Effect of the Aspect Ratio on the Transitional Structural behavior between plates and shells

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Abstract
There are several factor affecting the structural stiffness and rigidity of rectangular slab, including the slab thickness, the material modulus of elasticity, and the type of support. Any slight rise at the center of the slab will shift the behavior of the slab structurally, and it transforms the slab from two dimensional to three dimensional structural element. This transformation affects the values and the types of the internal stresses generated to resist imposed external loads. Mainly the shift will be from flexural stresses into compressive stresses. This study focuses on the effect of the aspect ratio on this transformation due to an induced rise at the center of the slab. If this rise is large enough, then the slab will behave as a shell. The increase in the rise pushes the flat square slab to become a hollow pyramid Finite element model has been developed to study the critical value of the rise that makes the stab shifts its behavior. Von Mises theory is used to calculate the critical stresses. The effect of the slab thickness on this critical rise is also studied. The effect of the slab aspect ratio is also taken into consideration. New mathematical relationships were developed based on the rigorous parametric study performed on slabs of different aspect ratios.

Keywords: transitional rise, hollow pyramid, shell element, plate element, aspect ratio

INTRODUCTION
The rectangular slab carries loads differently based on the relative dimensions of the length compared to the slab width. Square slab transmits the applied load to the support at all sides evenly for symmetrical and uniform loading conditions case. The change in the length of the slab compared to the width changes this harmony of transmitting the loads to the supporting system. This study focuses on this change in behavior due to relative dimensions of the slab length compared to its width. The aspect ratio ($\zeta$, zeta) is defined as follows:

$$\zeta = \frac{L}{W}$$

Where,

$L =$ slab length
$W =$ Slab width

For concrete slabs, the punching shear strength is a critical factor used in the design of flat slabs of long spans. AlNasra et al studied the introduction of new type of reinforcement to improve the punching shear strength of flat slabs. They conducted several experiments using steel swimmer bars. These steel bars main function is to improve the punching shear strength of the reinforced concrete square slabs. Ultimately the tested slabs failed by punching shear [1, 2].

Many researchers studied the behavior of slabs and shells separately. Most of these studies generated stresses and deformations of slabs and shells subjected to symmetrical loading. The finite element methods has been widely used nowadays due to the availability of sophisticated finite element software. It became much easier to analyze shell and plate elements of particular nonlinear material model using these software. Also production of computer hardware of high processing power made the finite element method and its application easily accessible and one of the favorite methods among engineers.

Circular plates were, in particular, the focus of many researches lately. Several numerical methods were exploited to solve for the internal stresses of the plates and shells and compare these results with experimental values. The maximum deflection at the center of a circular simply supported plate subjected uniformly distributed load can be expressed in the following the Equation (2), and Equation (3) [3, 4, 5, 6, 7].

$$\Delta_{\text{max}}= \left(\frac{w_o r^4 (5+\nu)}{64 D (1+\nu)}\right)$$

Where,

$w_o =$ uniformly distributed load
$r =$ radius of plate
$\nu =$ Poisson’s ratio
$D =$ flexural rigidity of plate.

The flexural rigidity of the plates is a function of the modulus of elasticity, plate thickness and Poisson’s ration. The rigidity of a plate may can be expresses as
D= E t³ / ( 12 (1 - v²) )

(3)

Where,
E = modulus of elasticity
t = thickness of plate

Then

\[ w_0 = \left[ \frac{64 D}{(1 + v)} \right] / (r^3 (5 + v)) \] \( \Delta_{\text{max}}/a \)

(4)

Equation (4) expresses the deflection at the center of the plates as a function of the applied uniformly distrusted load. This equation is known as small deflection theory. Many studies focused also on what became known as large deflection theory. The common type of loading used by researchers is the uniformly distributed load [8, 9, 10, and 11].

PLATE-SHELL THEORY

Equilibrium and compatibility are in the center of the finite element model to solve for stresses and deflections of a plate or a shell element. Proper selection of the type of the finite element along with the size of the finite element mesh are the two major factors affecting the accuracy of the theoretical results. The aspect ratio can be an additional factor that affects the accuracy and the structural behavior of the plate/shell element.

In order to determine the max stress in the plate transitioning to become a shell, the distortion energy theory is used. Von Mises theory is utilized in this study, which is based on the distortion energy. Von Mises theory applies for ductile solids that yield when the distortion energy reaches a specified critical stress value. The distortion energy, \( U \), in its simple form expressed in terms of the major principal stresses expressed per unit volume is shown in Equation (5).

\[ U = \frac{1 + v}{3E} \left( \sigma_1^2 - \sigma_2^2 \right) + \frac{(\sigma_2 - \sigma_3)^2}{2} + \frac{(\sigma_3 - \sigma_1)^2}{2} \]

(5)

Which is also can be expressed as

\[ U = \frac{1 + v}{3E} \sigma_\theta^2 \]

(6)

Where,
\( \sigma_{1,2,3} \) = principal stresses
\( \sigma_\theta \) = Von Mises stress

The simplified form of the Von Mises can be expressed in terms of the main principal stresses as shown in Equation (7)

\[ \sigma_\theta = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \]

(7)

The Von Mises stress in Equation (7) can be rewritten in its general form to include the shear stresses at any specified rotation in the three dimensional coordinate system

\[ \sigma_\theta = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}} \]

(8)

Where
\( \tau_{xy} \), \( \tau_{yz} \), \( \tau_{zx} \) = Shear in the specified plane

For two-dimensional state of stresses, where \( \sigma_3 = 0 \), the Von Mises collapses to

\[ \sigma_\theta = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2} \]

(9)

DISCUSSION AND PRESENTATION OF RESULTS

Four different slab sizes were used in this study. Several values of the slab aspect ratio were used to investigate the effect of the central rise on the stresses and the central deflection of the slabs. The critical rise at which there is substantial shift in the structural behavior is the focus of this research. The shift in the behavior from plate to shell is controlled by the value of the central rise as well as the thickness of the slab. The effect of the aspect ratio on the shift in behavior is presented. Aspect ratios of 1, 1.5, 2, and 4 were taken into consideration in this study. Three different slab thicknesses were also considered to study the effect of the slab thickness on the shift of the slab behavior. The slabs were subjected to both dead loads and live loads as follows

Dead load = self weight +5kN/m² as super imposed dead load
Live load = 3 kN/m²

Load combination = 1.4 (dead load) + 1.6 (live load), used for ultimate condition and 1.0 (dead load) + 1.0 (live load), used for service condition

The slabs are considered simply supported at their edges. The rise is applied at the center of the slab, and increases gradually by 2 cm increment. Figure (1) shows a typical rectangular slab subjected to central rise. For square slab, the central rise will transform the flat slab into hollow pyramid. Figure (2) through Figure (5) show the effect of the central rise on the value of the central deflection of the slab at a given slab aspect ratio.
Also, the effect of the central rise on the maximum Von Mises stress is studied for four different values of the aspect ratios. The effect of the aspect ratio on the lowest maximum stress is the focus of these graphs. Figure (6) through Figure (9) show the effect of the aspect ratio on the stress at a given value of the slab thickness.
Figure 6: Effect of the central rise on the max Von Mises stress of a square slab

Figure 7: Effect of the central rise on the max Von Mises stress of a slab of aspect ratio of 1.5

Figure 8: Effect of the central rise on the max Von Mises stress of a slab of aspect ratio of 2.0

Figure 9: Effect of the central rise on the max Von Mises stress of a slab of aspect ratio of 4.0

Figure 10 shows the effect of the slab thickness on the lowest maximum Von Mises stress ($\sigma_{L,\text{Max}}$) at the center of the slab. The figure shows that the increase in the slab thickness ($t$) decreases the lowest maximum Von Mises stresses in the slab. Also it can be noted that the lower the aspect ratio also decreases the lowest maximum Von Mises stress in the slab especially at the center of the slab. It can also be observed from the figure that the lower the aspect ratio the lower the stresses for a given slab thickness.

Regression analysis has been used to relate the lowest maximum Von Mises stress with the slab thickness and the slab aspect ratio. Equation (10) represents this relationship. This relationship is produced with $R^2$ value of 0.99, where $R$
is defined as the coefficient of determination of the regression analysis. This value indicates how close the data calculated to the derived function. The highest value $R^2$ can take is 1.0. The stress ($\sigma_{L,\text{Max}}$) is expressed in kPa, and the slab thickness is expressed in meter.

$$\sigma_{L,\text{Max}} = [-450 \zeta^2 + 6516 \zeta + 4498]e^{-11.2t}$$ (10)

Figure (11) relates the critical rise that generates the lowest maximum Von Mises stress with the slab thickness and the aspect ratio. The increase in the slab thickness increases the rise needed to produce low maximum Von Mises stress. The aspect ratio of the slab affects the value of the rise too. The lower the aspect ratio the lower the value of the rise needed to produce lowest maximum Von Mises stress.

The relationship between the critical rise of the slab thickness and the aspect ratio is also studied by generating a mathematical formula. Equation (11) represents a mathematical function that relates the critical rise ($R_c$) with the slab thickness and the aspect ratio. The square of the coefficient of determination, $R^2$, is 0.9995 for this particular mathematical formula.

$$R_c = (0.4 \zeta + 4.2)t + 0.0066 \zeta^2 - 0.0464 \zeta + 0.083$$ (11)

Where $R_c$ and $t$ are measured in meter.

Figure (12) shows a different relationship between the slab thickness and its effect on the critical rise at a given aspect ratio. The increase in the aspect ratio substantially increases the critical rise. The increase in the slab thickness will also increase the value of the critical rise

CONCLUSIONS

Both the slab thickness and the aspect ratio play significant role in determining the values of the state of stresses in the slab. The increase in the aspect ratio increases the stresses in the slab. Also the increase in the slab thickness increases the critical rise, the rise that produces lowest maximum Von Mises stress. Large aspect ratio makes the analysis and design of the slabs as plate or shell inefficient. Providing a small rise at the center of a rectangular slab reduces the stresses and makes the slab more economical. The central deflection of the rectangular slabs reduces by two major factors; the increase in the slab thickness and the increase in the central rise. It is more economical to reduce the central deflection by inducing central rise in a rectangular slab. The rectangular slabs shift its behavior with the increase in the central rise.

REFERENCES


