Pergamon

Int. Comm. Heat Mass Transfer, Vol. 29, No. 8, pp. 1159-1167, 2002

0735–1933/02/$–see front matter

PII: S0735-1933(02)00444-X

EFFICIENCY OF MILLER ENGINE AT MAXIMUM POWER DENSITY

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(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

Thermodynamic analysis of an ideal air-standard Miller cycle is presented in this paper. The paper outlines the effect of maximizing power density on the performance of the cycle efficiency. The power density defined as the ratio of the power to the maximum cycle specific volume. Although the efficiency of Atkinson and Joule-Brayton cycles at maximum power density is greater than that of Miller cycle, it is important to note that the total cycle volume and pressure ratio at maximum power density of Miller is smaller. The results obtained from this work can be helpful in the thermodynamic modeling and in evaluation of Miller engines over Atkinson and Joule-Brayton engines.

Introduction

The Miller cycle, named after its inventor R.H. Miller, has an expansion ratio greater than its compression ratio [1,2]. The Miller cycle, shown in Figure 1, is considered to be a modern modification of the Atkinson cycle (i.e., complete expansion cycle). The cycle that holds his name is generally one of the more-complete-expansion cycles. The compression ratios of spark ignition, gasoline-fueled engines are limited by knock and fuel quality in the range of 8 to 11, depending upon cylinder design [3]. In conventionally constructed engines, the compression ratio is also the expansion ratio. Therefore, whatever limits compression also limits the extent to which the high-pressure combustion gases can be expanded, and the work is extracted from them. Thus, the scheme is to greatly raise the compression ratio (and consequently, the expansion ratio), in order to avoid knocking by shortening the intake stroke. This shortening can be achieved either by fiddling with the intake valve timing, or by introducing a controlled intake throttling to accomplish the same goal; limiting the intake mass flow so that compression at the new, very high ratio does not result in detonation [4].

Optimization and performance analysis can be applied using finite time thermodynamics techniques. They are used to study performance of various air-standard power cycles [5-9]. For example, Chen *et. al.* [6] and Sahin *et. al.* [7] examined Atkinson cycle and Joule-Brayton cycle respectively at maximum power density. Both studies found that the efficiency at the maximum power density is greater than that at the maximum power. In this paper the study will be extended to a Miller cycle. A comparison between the present work and Chen et al. results on Atkinson engine) [7] and Sahin et al. results on Joule-Brayton engine (will take the symbol J-B) will be presented in this paper.

A maximization of the power density (the ratio of the maximum power to the maximum specific volume in the cycle) is taking into consideration the engine size instead of just maximizing the power. Including the engine size in the calculation of the engine performance is a very important factor from an economical point of view.

In this paper different parameters affecting cycle performance and net work output at maximum power density will be considered. The obtained results will be presented as performance characteristic curves for the Miller cycle using numerical examples and compared with Atkinson and Joule-Brayton (J-B) cycles.

Thermodynamic Analysis

Figure 1 presents pressure-volume (*P-V*) and temperature-entropy (*T-S*) diagrams for the thermodynamic processes performed by an ideal air-standard Miller cycle [2,5]. All five processes are reversible. Process 1-2 is an adiabatic (isentropic) compression; process 2-3 is a heat addition at a constant pressure; process 3-4 adiabatic (isentropic) expansion; process 4-5 is heat rejection at a constant volume; finally, process 5-1 is a compression process at a constant pressure. Assuming constant specific heats, the power output, per unit mass of the working fluid, can be written in the form:

$$W = C_v\left(\left(T_3 - T_2\right) - \left\{\left(T_4 - T_5\right) + k\left(T_5 - T_1\right)\right\}\right) \quad (1)$$

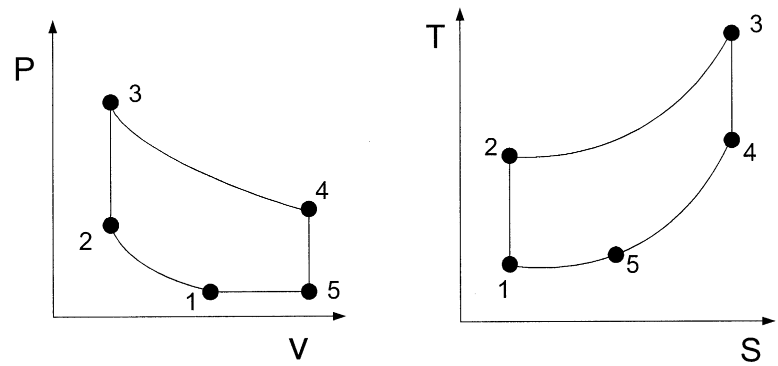

FIG. 1
Miller's cycle *P-V* and *T-S* diagrams.

where $k = C_p/C_v$ is the specific heat ratio, and C_p and C_v is the constant-pressure and constant-volume specific heats, respectively; and T_1, T_2, T_3, T_4 and T_5 are the absolute temperatures at states 1, 2, 3, 4 and 5, respectively. The power density (P) is defined as the output power per maximum specific volume (v_5) in the cycle and is given by the formula

$$P = \frac{C_v}{v_5} T_1 \left[\left(\frac{T_3}{T_1} - \frac{T_2}{T_1} \right) - \left(\frac{T_4}{T_1} - \frac{T_5}{T_1} \right) - k \left(\frac{T_5}{T_1} - 1 \right) \right] \tag{2}$$

From the *T-S* diagram of Miller cycle the following equation can be derived

$$\frac{T_3}{T_2} = \left(\frac{T_4}{T_1} \right) \left(\frac{T_5}{T_1} \right)^{k-1} \tag{3}$$

The following dimensionless parameters are defined in terms of the Miller cycle temperatures:

$\theta = \frac{T_2}{T_1}$, $\tau = \frac{T_3}{T_1}$, $\delta = \frac{T_4}{T_1}$ and $\sigma = \frac{T_5}{T_1}$ substituting in equation (3) yields

$$\delta = \frac{\tau}{\theta} \sigma^{1-k} \tag{4}$$

Using the above parameters and equation (4), the power density in equation (2) and the output power in equation (1) can be written in terms of θ, τ, δ and σ as

$$P = \frac{C_v T_1}{v_1} \left((\tau - \theta + k) \left(\frac{1}{\sigma} \right) - \frac{\tau}{\theta} \sigma^{-k} - k + 1 \right) \tag{5}$$

$$W = C_v T_1\left(\tau - \theta - \frac{\tau}{\theta}\sigma^{1-k} + \sigma - k(\sigma - 1)\right) \tag{6}$$

Finally the thermal efficiency of Miller cycle will be

$$\eta = 1 - \frac{\tau\sigma^{1-k}}{\theta(\tau - \theta)} + \frac{\sigma}{\tau - \theta} - \frac{k(\sigma - 1)}{\tau - \theta} \tag{7}$$

Power Density Optimization

For a given τ and σ the power density can be maximized by taking the derivative of P (equation 5) with respect to θ and equating it to zero ($dP/d\theta = 0$) gives

$$\theta_p = \sqrt{\tau\sigma^{1-k}} \tag{8}$$

where θ_p is the optimal power density isentropic temperature ratio. Thus the maximum power density will be

$$P_{max} = \frac{C_v T_1}{v_1}\left[\left\{\tau - \sqrt{\tau\sigma^{1-k}} + k\right\}\left(\frac{1}{\sigma}\right) - \left(\frac{\tau}{\sqrt{\tau\sigma^{1-k}}}\right)\sigma^{-k} - k + 1\right] \tag{9}$$

Also, the cycle efficiency at the maximum power density point is

$$\eta_p = 1 - \frac{\sigma^{1-k}}{\sqrt{\tau\sigma^{1-k}} - \sigma^{1-k}} + \frac{\sigma - k(\sigma - 1)}{\tau - \sqrt{\tau\sigma^{1-k}}} \tag{10}$$

For a given τ and σ the output power can be maximized by taking the derivative of W with respect to θ and equating it to zero ($dW/d\theta = 0$) gives

$$\theta_w = \sqrt{\tau\sigma^{1-k}} \tag{11}$$

It is worthy to note that the maximum power density and the maximum output power occur at the same point (i.e. $\theta_p = \theta_w$) which is different from the work done on Aktinson engine [7] and Joule-Brayton engine (J-B) [8]. Then the maximum output power will be

$$W_{max} = C_v T_1\left(\tau - 2\sqrt{\tau\sigma^{1-k}} + \sigma - k(\sigma - 1)\right) \tag{12}$$

The cycle efficiency at the maximum output power point will be identical to equation (10). Since the maximum power density occurs at the same point, as in the case of maximum output power, then, the ratio of the power density to the maximum power density (P/P_{max}) is identical to the ratio of the output power to the maximum output power (W/W_{max}). This can be expressed in the following formula

$$\frac{W}{W_{\max}} = \frac{P}{P_{\max}} = \left(\tau - \theta - \frac{\tau}{\theta}\sigma^{1-k} + \sigma - k(\sigma - 1)\right) / \left(\tau - 2\sqrt{\tau\sigma^{1-k}} + \sigma - k(\sigma - 1)\right) \tag{13}$$

In order to consider the total engine size, including the compressor and other parts, the cycle displacement volume ($v_5 - v_2$) at the maximum power density point (at $\theta = \theta_p$) is normalized by the initial volume (v_1) which will be considered and will have the form

$$\frac{v_5 - v_2}{v_1} = \sqrt{\sigma}(\sqrt{\sigma} - \sqrt{\tau^{(\frac{1}{1-k})}}) \tag{14}$$

The cycle pressure ratio (P_3/P_1) at $\theta = \theta_p$ is defined as

$$\frac{P_3}{P_1} = \left(\frac{\tau}{\theta_p}\right)(\theta_p)^{k/(k+1)} = \tau\left(\sqrt{\tau\sigma^{1-k}}\right)^{1/(k-1)} \tag{15}$$

Performance Comparison

The derived formulae above are used and plotted in order to compare Miller engine with Atkinson engine results [7] and Joule-Brayton results [8] as shown in Figures 2 through 6. The following constants and range of parameters are selected:

$k = 1.4 \quad \sigma = 1.1 \quad \tau = 1$ to 6

By varying isentropic temperature ratio (θ) or varying thermal efficiency (η) and given value for cycle temperature ratio ($\tau = 4$) the normalized power density P/P_{max} and normalized power W/W_{max} are plotted in Figure 2

As it was presented earlier, the thermal efficiency of Miller cycle at maximum power density is equal to that at maximum power since in case of Miller cycle ($\theta_p = \theta_w$). On the other hand, the thermal efficiency at maximum power density (η_p) is greater than the efficiency at maximum power (η_w) for Atkinson and Joule-Brayton cycles. The thermal efficiency of Miller cycle is less than that of Atkinson and J-B cycles. The thermal efficiency at maximum power density of Miller is very close to that of Atkinson and J-B at maximum power condition.

Figure 3 shows the variation of the thermal efficiency with cycle temperature ratio (τ). The thermal efficiency at maximum power density of Atkinson and Joule-Brayton cycles are very similar and larger than that at maximum power for all values of τ. The thermal efficiency of

Miller at maximum power is very close to those of Atkinson and J-B at maximum power condition.

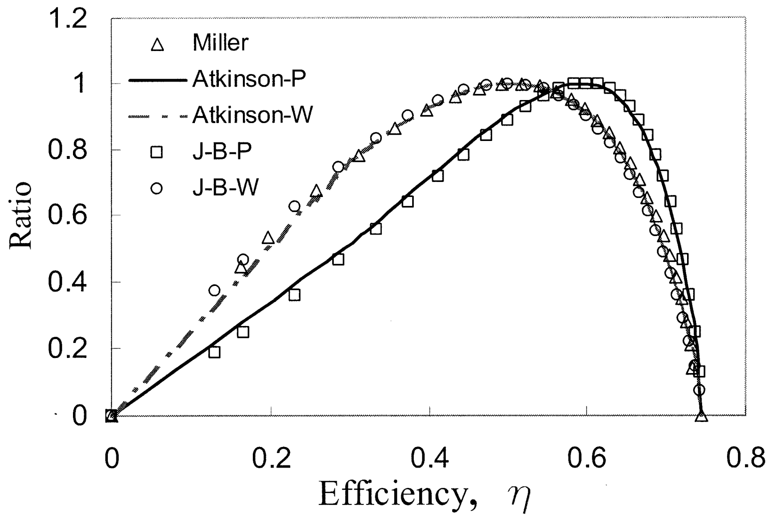

FIG. 2

Variation of normalized power and normalized power density with thermal efficiency at τ = 4.0
Ratio: Miller is P/Pmax or W/Wmax of Miller; Atkinson-P is P/Pmax of Atkinson; Atkinson-W is W/Wmax of Atkinson; J-B-P is P/P/max of Joule-Brayton and J-B-W is W/Wmax of Joule-Brayton cycle

Figure 4 shows the maximum cycle volume normalized by the initial specific volume (atmospheric condition). This represents a normalized total engine size. As shown in Figure 4 Miller engine at maximum power density condition has the smallest engine size at the same cycle temperature ratio and the increase in the volume ratio after about τ = 2 is very small compared to the other engines.

Figure 5 shows the normalized cycle pressure ratio. It can be concluded from this Figure that engines working at J-B maximum power condition will have the lowest pressure ratio followed by Atkinson at maximum power point. Engines that are working at Atkinson maximum power density condition will have the maximum pressure ratio even though they have the maximum thermal efficiency as it was illustrated in Figure 3. Finally, the variations of isentropic temperature ratio at maximum power and maximum power density (θ_w and θ_p) with temperature ratio for all cycles are shown in Figure 6. It is clear from this Figure that up to τ around 2 all θ_w and θ_p are similar for all cycles.

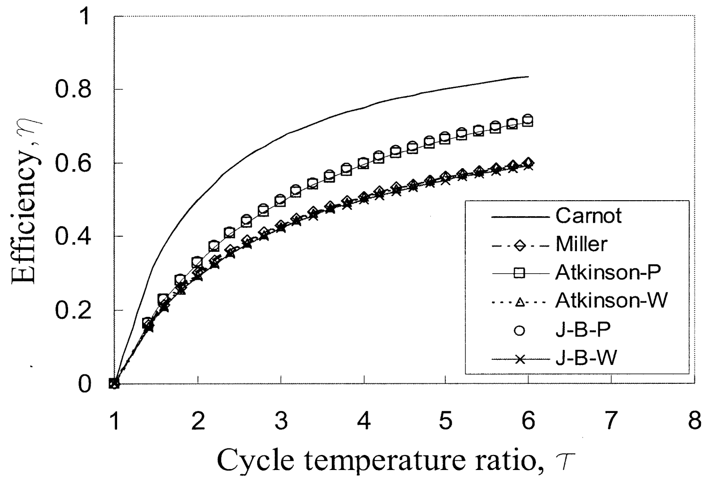

FIG. 3
Efficiency versus cycle temperature ratio for different cycles. Legends are: Carnot is η for Carnot; Miller: η at $\theta = \theta_p$ or $\theta = \theta_w$; Atkinson-P: η at $\theta = \theta_p$; Atkinson-W: η at $\theta = \theta_w$; J-B-P: η $\theta = \theta_p$ of Joule-Brayton and J-B-W: η at $\theta = \theta_w$ of Joule-Brayton cycle.

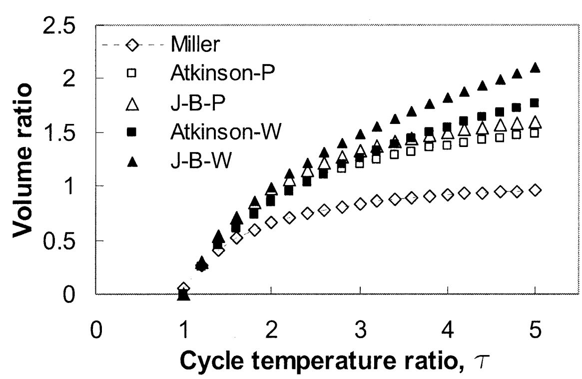

FIG. 4
Volume ratio variation with cycle temperature ratio

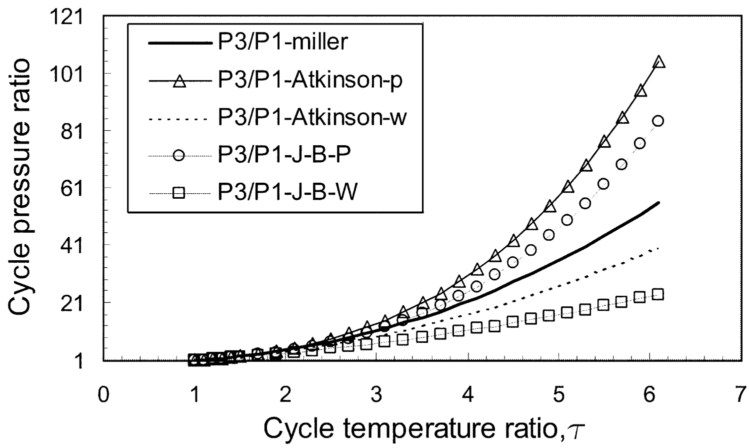

FIG. 5
Cycle pressure ratio versus cycle temperature ratio at maximum power and maximum power density (Legends: P means value is taken at $\theta = \theta_p$; W means value is taken at $\theta = \theta_w$)

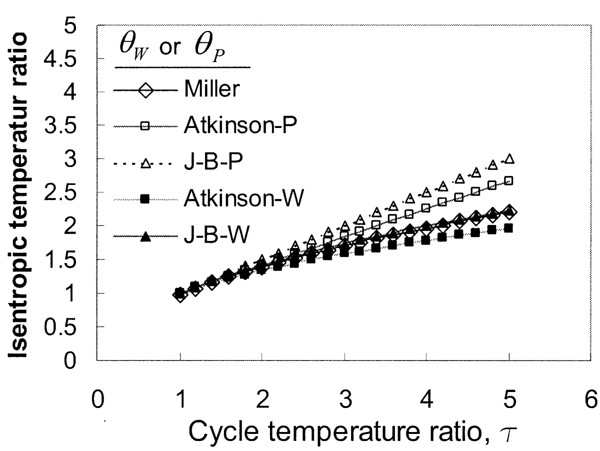

FIG. 6
Variation of the isentropic temperature ratio with cycle temperature ratio at maximum power and at maximum power density

Conclusions

The Miller cycle maximum power density and maximum power are identical. However, Atkinson and Joule-Brayton maximum power density and maximum power points are different. To have optimum design parameters all the performance characteristics, advantages and disadvantages discussed above must be taken all together, for example, although an Atkinson engine designed using parameters at the maximum power density has higher thermal efficiency, than for an engine designed using Miller cycle at maximum power density but it requires higher pressure ratio (Figure 5) and higher specific volume ratio (Figure 4). Therefore, an optimization technique must be made by taking all parameters into account including the thermal performance, engine size and investment cost.

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Received July 14, 2002