Entropy generation and MHD natural convection of a nanofluid in an inclined square porous cavity: Effects of a heat sink and source size and location

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Keywords:
Entropy generation
Nusselt number
Hartmann number
Thermal performance criteria
Heat sink and source

Abstract

The effects of a heat sink and the source size and location on the entropy generation, MHD natural convection flow and heat transfer in an inclined porous enclosure filled with a Cu-water nanofluid are investigated numerically. A uniform heat source is located in a part of the bottom wall, and a part of the upper wall of the enclosure is maintained at a cooled temperature, while the remaining parts of these two walls are thermally insulated. Both the left and right walls of the enclosure are considered to be adiabatic. The thermal conductivity and the dynamic viscosity of the nanofluid are represented by different verified experimental correlations that are suitable for each type of nanoparticle. The finite difference methodology is used to solve the dimensionless partial differential equations governing the problem. A comparison with previously published works is performed, and the results show a very good agreement. The results indicate that the Nusselt number decreases via increasing the nanofluid volume fraction as well as the Hartmann number. The best location and size of the heat sink and the heat source considering the thermal performance criteria and magnetic effects are found to be D = 0.7 and B = 0.2. The entropy generation, thermal performance criteria and the natural heat transfer of the nanofluid for different sizes and locations of the heat sink and source and for various volume fractions of nanoparticles are also investigated and discussed.

1. Introduction

Addition of a small amount of high thermal conductivity nanoparticles by volume to common base or working fluids (thus forming nanofluids) is a novel mechanism which may resolve the challenge of improving the heat transfer in several industries, such as cooling electronic designs, drying devices, solar collectors etc. Changing the flow regime and using porous materials is another way to enhance the heat transfer in various systems. Thus, the heat transfer of nanofluids in porous media is a new subject that needs more study and consideration by authors. Some works focusing on the natural convection heat transfer of nanofluids in porous media have been recorded. The problem of free convection past a vertical plate using a nanofluid was investigated by Nield and Kuzentov [1]. Gorla and Chamkha [2] examined a free convection boundary layer over a non-isothermal flat plate embedded in a porous medium.
The natural convection of a nanofluid on a sphere in a porous media was studied by Chamkha et al. [3]. Cheng [4] repeated the studies of Gorla and Chamkha [2] and Chamkha et al. [3], but for a truncated cone. However, the sole work on nanofluids in a cavity-filled by porous media is that of Sun and Pop [5], where they considered a triangular enclosure heated by a wall heater filled with a porous medium and saturated with three different nanofluids. Chamkha and Ismael [6] studied the conjugate heat transfer in a porous cavity filled with nanofluids and heated by a triangular thick wall. Sheremet and Pop [7] studied the conjugate natural convection of a nanofluid in a porous cavity. They found that the local Nusselt number is an increasing function of the Rayleigh number. Sheremet et al. [8] also extended their investigation for three-dimensional natural heat transfer. The effect of nanoparticles on natural convection in a porous media was studied by Bourantas et al. [9]. The natural convection boundary-layer flow adjacent to a vertical cylinder embedded in a thermally stratified nanofluid-saturated non-Darcy porous medium was studied by Rashad et al. [10]. They also investigated the effect of uniform lateral mass flux on the non-Darcy natural convection of a non-Newtonian fluid along a vertical cone embedded in a porous medium filled with a nanofluid [11]. Sheikholeslami and Rokni [12] investigated the effect of melting heat transfer on a nanofluid flowing over a porous stretching plate under a magnetic field. A revised model for the Darcy-Forchheimer flow of a Maxwell nanofluid subject to a convective boundary condition was presented by Muhammad et al. [13].

All of the above-mentioned investigations are based on the first-law analysis. Lately, the second law-based works have acquired much concern for analyzing thermal systems. Entropy generation is applied as a criterion to assess the rendering of thermal systems. The study of the exergy employment and the entropy generation has been one of the fundamental aims in studying thermal systems. Bejan [14–16] pointed out the several causes of entropy generation in applied thermal engineering. It is known that the generation of entropy decreases the effectiveness of an engineering system. Therefore, it is of paramount importance to study the irreversibility of heat transfer and fluid friction operation. Abolbashari et al. [17] studied the entropy generation for a Casson nanofluid flow induced by a stretching surface with the homotopy analyzing method. Armaghani et al. [18,19] studied the entropy generation and natural heat transfer of a nanofluid in C-shaped and baffled L-shaped cavities, respectively. The entropy generation due to the electrical unsteady natural MHD flow of a nanofluid and heat transfer was studied numerically by Daniel et al. [20].

There are just very few works that study the entropy generation due to natural convection of nanofluids in porous media. The entropy generation analysis due to a rotating porous disk under a magnetic field and using a nanofluid as a working fluid was done by Rashidi et al. [21]. The entropy generation on MHD Casson nanofluid flow over a porous stretching/shrinking surface was investigated by Qiu et al. [22]. Recently, Ismael et al. [23] investigated the entropy generation due to conjugate free convection in a square domain. They suggested a novel gauge for the evaluation of the thermal performance. Armaghani et al. [24] studied the entropy generation and natural convection of nanofluids in an inclined porous media-layered cavity. They used the thermal performance criteria for obtaining the best value of the porous layer, nanofluid volume fraction and other parameters. Al-Azamiy [25]
studied numerically the heat transfer, fluid flow and entropy generation through a multi-layer cavity. It was found that by increasing the Darcy number, the average Nusselt number decreases. The effects of a heat sink and source and entropy generation on the MHD mixed convection of a Cu-water nano-fluid in a lid-driven square porous enclosure with partial slip was studied by Chamkha et al. [26].

The above literature review shows that the effects of a heat sink and the source location and size on the natural convection heat transfer and entropy generation of a nano-fluid in an inclined porous cavity in the presence of a magnetic field has not been investigated yet. Hence this work is the focus of the current article. It is considered that this investigation will contribute in developing the thermal effectiveness in several engineering devices.

2. Geometry of the problem and the mathematical model

The schematic diagram of the current study with the coordinate system and boundary conditions is depicted in Fig. 1. The model is of two-dimensional natural convection inside an inclined square cavity of length \( H \) filled with a Cu-water nano-fluid-saturated porous media. The coordinates \( x \) and \( y \) are chosen such that \( x \) measures the distance along the bottom horizontal wall, while \( y \) measures the distance along the left vertical wall, respectively. The angle of inclination of the enclosure from the horizontal in the counterclockwise direction is denoted by \( \Phi \). A magnetic field with strength \( B_0 \) is applied on the left side of the enclosure in the positive horizontal direction. A heat source with a constant volumetric rate \( (Q') \) is located on a part of the bottom wall, and a part of the upper wall of the enclosure is maintained at a cooled temperature \( T_c \) with length \( b \), while the other remaining parts of these two walls are thermally insulated. Both the left and right walls of the enclosure are considered to be adiabatic. The nano-fluid used in the analysis is assumed to be incompressible, laminar and generates heat at a uniform rate \( Q_0 \), and the base fluid (water) and the solid spherical nanoparticles (Cu) are in thermal equilibrium. The thermo-physical properties of the base fluid and the nanoparticles are given in Table 1. The thermo-physical properties of the nano-fluid are assumed constant except for the density variation, which is determined based on the Boussinesq approximation Fig. 2.

Under the above assumptions and by considering the works presented in [26–29] the governing equations representing the continuity, momentum and energy equations take the following form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{u}{\partial x} + \frac{v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{K} u \right) + \frac{(\rho \beta_{nf})}{\rho_{nf}} g (T - T_c) \sin \Phi, \tag{2}
\]

\[
\frac{u}{\partial x} + \frac{v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{1}{K} v \right) + \frac{(\rho \beta_{nf})}{\rho_{nf}} g (T - T_c) \cos \Phi - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} v, \tag{3}
\]

\[
\frac{u}{\partial x} + \frac{v}{\partial y} = \frac{1}{\rho_{nf}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{\left( \rho c_{p,nf} \right)} Q_0 (T - T_c), \tag{4}
\]

where \( u \) and \( v \) are the velocity components along the \( x \)- and \( y \)- axes respectively, \( T \) is the fluid temperature, \( p \) is the fluid pressure, \( g \) is the acceleration due to gravity, \( K \) is the permeability of the porous medium, \( B_0 \) is the magnetic field strength, \( Q_0 \) is the heat generation \( (Q_0 > 0) \) or absorption \( (Q_0 < 0) \) coefficient, \( \rho_{nf} \) is the density, \( \mu_{nf} \) is the dynamic viscosity, and \( \nu_{nf} \) is the kinematic viscosity.
The boundary conditions are:

\[ u = v = 0, \quad 0 \leq x \leq H, \quad 0 \leq y \leq H, \]

\[ \frac{\partial T}{\partial y} = -\frac{q'}{k_{nf}}, \quad (d - 0.5b) \leq x \leq (d + 0.5b), \]

and \[ \frac{\partial T}{\partial y} = 0, \quad \text{otherwise on wall} \quad y = 0, \]

\[ \frac{\partial T}{\partial x} = 0 \quad \text{on walls} \quad x = 0, H, \]

\[ T = T_c, \quad (d - 0.5b) \leq x \leq (d + 0.5b), \]

and \[ \frac{\partial T}{\partial y} = 0, \quad \text{otherwise on wall} \quad y = H. \]  

\[ (5) \]

2.1. Thermo-physical properties of the nanofluid

The effective density and heat capacitance of the nanofluid are given as [30]

\[ \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p. \]  

\[ (6) \]

\[ \left(\rho c_p\right)_{nf} = (1 - \phi)\left(\rho c_p\right)_f + \phi\left(\rho c_p\right)_p. \]  

\[ (7) \]

The thermal expansion coefficient of the nanofluid can be determined according to Khanafer et al. [30] by:

\[ (\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_p. \]  

\[ (8) \]

The thermal diffusivity, \( \alpha_{nf} \), of the nanofluid is defined by Abu-Nada and Chamkha [31] as:

\[ \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}. \]  

\[ (9) \]

In Eq. (9), \( k_{nf} \) is the thermal conductivity of the nanofluid which for spherical nanoparticles according to the Maxwell-Garnetts model [32] is given by:

![Fig. 2. Comparison of the current isotherms (right) and those obtained by Aminossadati and Ghasemi [27] (left) at B = 0.4 and Ra = 10^5, \( \phi = 0.1 \), \( D = 0.5 \).](image)
Following the study of Ghalambaz et al. [33], the effective thermal conductivity of the porous medium and the nanofluid is evaluated using:

\[ k_{\text{eff, sf}} = \varepsilon k_s + (1 - \varepsilon)k_f \]  \hspace{1cm} (10a)

where \( k_s \) is the solid thermal conductivity and \( \varepsilon \) is the porosity of the porous medium. Accordingly, the effective thermal conductivity of the base fluid and the porous medium \( (k_{\text{eff, bf}}, f) \) can be evaluated using the following equation:

\[ k_{\text{eff, bf}} = \varepsilon k_f + (1 - \varepsilon)k_s \]  \hspace{1cm} (10b)

These equations are valid when the thermal conductivity of the solid and the fluid are close, as discussed by Prasad et al. [34]. Hence, in representing the results, the thermal conductivity of the fluid and the porous medium have been considered to be very close.

The effective thermal diffusivity of the nanofluid and the porous matrix is also introduced as:

\[ \kappa_{\text{eff, sf}} = \frac{k_{\text{eff, sf}}}{\rho c_p f} \]  \hspace{1cm} (10c)

The effective dynamic viscosity of the nanofluid based on the Brinkman model [35] is given by

\[ \mu_{\text{eff}} = \frac{\mu_f}{(1 - \phi)^3} \]  \hspace{1cm} (11)

The effective electrical conductivity of the nanofluid was presented by Maxwell [32] as

\[ \frac{\sigma_{\text{eff}}}{\sigma_f} = 1 + \frac{3(\gamma - 1)\phi}{(\gamma + 2)(\gamma - 1)\phi} \]  \hspace{1cm} (12)

where \( \gamma = \frac{\sigma_f}{\sigma_p} \).

### 2.2. Dimensionless forms of equations

The following non-dimensional parameters are introduced:

\[ X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha_f}, \quad V = \frac{vH}{\alpha_f}, \quad P = \frac{\rho H^2}{\alpha_f}, \quad \theta = \frac{T - T_c}{\Delta T}, \quad \beta = \frac{\Delta T}{\Delta T}, \quad \Delta T = \frac{qH}{k_f}, \quad Ec = \frac{\kappa^2}{H^3(\alpha_f)\Delta T}. \]  \hspace{1cm} (13)

Substituting Eq. (13) into Eqs. (1)–(5) yields the following dimensionless equations:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \]  \hspace{1cm} (14)

\[ U \frac{\partial^2 U}{\partial X^2} + V \frac{\partial^2 U}{\partial Y^2} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\alpha_{\text{eff}}}{\alpha_f} \right) \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{Da} \right) + \left( \frac{\rho \beta_{\text{eff}}}{\rho_f \beta_f} \right) RaPr \sin \Phi \theta, \]  \hspace{1cm} (15)

\[ U \frac{\partial^2 V}{\partial X^2} + V \frac{\partial^2 V}{\partial Y^2} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\alpha_{\text{eff}}}{\alpha_f} \right) \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} - \frac{V}{Da} \right) + \left( \frac{\rho \beta_{\text{eff}}}{\rho_f \beta_f} \right) RaPr \cos \Phi \theta - Ha^2 Pr \left( \frac{\sigma_{\text{eff}}}{\sigma_f} \right) \left( \frac{\beta_f}{\beta_{\text{eff}}} \right) V, \]  \hspace{1cm} (16)

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{\text{eff}}}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \left( \frac{\rho \beta_{\text{eff}}}{\rho_f \beta_f} \right) Q\theta, \]  \hspace{1cm} (17)

where

\[ Pr = \frac{\nu_f}{\alpha_f}, \quad Ra = \frac{g \beta_f \Delta T H^3}{\nu_f \alpha_f}, \quad Ha = B_0 H \sqrt{\alpha_f / \mu_f}, \quad Da = K / H^2. \]

<table>
<thead>
<tr>
<th>Ra</th>
<th>Aminossadati and Ghasemi [27]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>5.451</td>
<td>5.450</td>
</tr>
<tr>
<td>$10^4$</td>
<td>5.474</td>
<td>5.475</td>
</tr>
<tr>
<td>$10^5$</td>
<td>7.204</td>
<td>7.204</td>
</tr>
<tr>
<td>$10^6$</td>
<td>14.014</td>
<td>14.014</td>
</tr>
</tbody>
</table>

Table 2
Comparison of the average Nusselt number $Nu_{av}$ for $B = 0.4$, $\phi = 0.1$, $D = 0.5$. 

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The above parameters are the Prandtl number, Rayleigh number, magnetic number and the Darcy number, respectively.

The dimensionless boundary condition for Eqs. (14)-(17) are written as:

\[
\begin{align*}
U &= V = 0, \quad 0 \leq X \leq 1, \quad 0 \leq Y \leq 1, \\
\frac{\partial \theta}{\partial Y} &= -\frac{k_f}{k_{nf}}, \quad (D - 0.5B) \leq X \leq (D + 0.5B) \quad \text{and} \quad \frac{\partial \theta}{\partial Y} = 0, \quad \text{otherwise on wall} \quad Y = 0, \\
\frac{\partial \theta}{\partial X} &= 0, \quad \text{on walls} \quad X = 0, \quad H, \\
\theta &= 0, \quad (D - 0.5B) \leq X \leq (D + 0.5B) \quad \text{and} \quad \frac{\partial \theta}{\partial Y} = 0, \quad \text{otherwise on wall} \quad Y = 1.
\end{align*}
\]  

(18)

The local Nusselt number is defined as

\[
Nu_s = \frac{1}{(\theta)_{\text{heat source wall}}},
\]

(19)

and the average Nusselt number is defined as

\[
Nu_{\text{avg}} = \frac{1}{B} \int_{D-0.5B}^{D+0.5B} Nu_s \, dX.
\]

(20)
2.3. Governing equation for entropy generation

The entropy generation in the flow field is caused by the non-equilibrium flow imposed by boundary conditions. According to Mahmud and Fraser [36], the dimensional local entropy generation can be expressed by
In Eq. (21), the first term represents the dimensional entropy generation due to heat transfer (sh), the second term represents the dimensional entropy generation due to fluid fraction irreversibility resulting from Darcy dissipation and viscous dissipation (sv), and the third term is the dimensional entropy generation due to the effect of the magnetic field (sm). By using the dimensionless parameters presented in Eq. (13), the expression for the non-dimensional entropy generation, \( S \) can be written as

\[
S = s \cdot \frac{H^2}{k_f} = \left( \frac{k_{ef}}{k_f} \right) \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\mu_{ef}}{\mu_f} \right) \left( \frac{1}{\alpha} (u^2 + v^2) + \frac{2}{\alpha} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) + \left( \frac{\sigma_{ef}}{\sigma_f} \right) B_0^2 v^2.
\]

(21)

Fig. 6. Variation of the average Nusselt number for Cu-water at \( Ha = 10, D = 0.5, Ra = 10^3, Q = 1, \Phi = 15^\circ \) (a) \( Nu_m \) & (b) \( Nu_m^+ \).

where \( C_T = \frac{\Delta T}{\alpha} \) is the difference temperature and \( Ec = \frac{\sigma^2 T}{\mu^2 (\sigma_D^2 + \Delta T)} \) is the Eckert number.

Here \( Sh, Sv \) and \( Sj \) are the dimensionless local entropy generation rate due to heat transfer, fluid fraction and Joule heating, respectively.

In order to present the effect of nanoparticles, the magnetic field and the difference of the temperature on the average Nusselt number and the total entropy generation, the following Nusselt ratios, total entropy generation ratios and the thermal performance criteria ratios are defined:
Fig. 7. Variation of the average total entropy generation ((a) $S$ & (b) $S^+$) for Cu-water for $\varphi = 0.005$, $Ha = 10$, $D = 0.5$, $Ra = 10^5$, $Q = 1$, $\Phi = 15^\circ$.

Fig. 8. Variation of global entropy to average Nusselt number at $\varphi = 0.005$, $Ha = 10$, $D = 0.5$, $Ra = 10^5$, $Q = 1$, $\Phi = 15^\circ$.

$$Nu^* = \frac{Nu_m}{(Nu_m)_{H=0}} \quad \text{and} \quad Nu^{*+} = \frac{Nu_m}{(Nu_m)_{H=0}},$$

(23)

$$S^* = \frac{S}{(S)_{H=0}} \quad \text{and} \quad S^{*+} = \frac{S}{(S)_{H=0}},$$

(24)
Fig. 9. Variation of the average Nusselt number for Cu-water at $\varphi = 0.005$, $D = 0.5$, $Ra = 10^5$, $Q = 1$, $\Phi = 15^\circ$.

Fig. 10. Variation of global entropy to average Nusselt number at $\varphi = 0.005$, $D = 0.5$, $Ra = 10^5$, $Q = 1$, $\Phi = 15^\circ$.

Fig. 11. Variation of the average Nusselt number for Cu-water at $\varphi = 0.005$, $Ha = 10$, $D = 0.5$, $Ra = 10^5$, $Q = 1$.

\[ e^+ = \frac{S^+}{Nu_m} \quad \text{and} \quad e^{++} = \frac{S^{++}}{Nu_m^{++}}. \]
3. Numerical solution and validation

In this work the numerical algorithm applied to solve the dimensionless governing Eqs. (14)–(17) is founded on the finite difference methodology. The central difference quotients have been utilized to approximate the second derivatives in both the X- and Y-directions. Hence the obtained discretized equations have been solved utilizing the successive under-relaxation (SUR) method. The solution procedure is iterated until the convergence norm is satisfied:

$$\sum_{i,j} |\chi_{i,j}^{\text{new}} - \chi_{i,j}^{\text{old}}| \leq 10^{-7},$$

where $\chi$ is the general dependent variable. The appropriate value of the relaxation parameter is found to be equal 0.7. In addition, Eq. (17) is computed by applying the trapezoidal rule. The numerical method is implemented in the FORTRAN software. The accuracy tests are made for grid independence using the finite difference method with four sets of grids: 41 × 41, 61 × 61, 81 × 81, and 101 × 101. A good agreement is found between the (81 × 81) and (101 × 101) grids, so the numerical computations are carried out for (81 × 81) grid nodal points.

In order to check the accuracy of the present method, the obtained results are compared in special cases ($B = 0.4$, $\phi = 10\%$, $D = 0.5$.) with the results obtained by Aminossadati and Ghasemi [27]. These comparisons are presented clearly in Fig. 2 and Table 2 in terms of the average Nusselt number at the heat source. A very good agreement is found between the results.

Also, taking into account the MHD influence, Sheikholeslami et al. [37] have examined the natural convection of Cu-water nanofluids in a square cavity. In the investigation of Sheikholeslami et al. [37] the upper wall of the cavity was well insulated, while the vertical side walls were at a constant temperature $T_c$. The lower wall was partially heated, which was at a constant temperature $T_h$. According to the boundary condition and notation of Sheikholeslami et al. [37], the Nusselt number was 9.429 when $Ra = 10^5$, $\varepsilon = 0.8$ (the bottom flash element length), $\phi = 0.04$ and $Ha = 100$. Here we obtained the Nusselt number as 9.4286, which exhibits a very good agreement with the literature value.

4. Results and discussion

The selective results are presented for the fixed values of $Ec = 10^{-3}$, $Da = 10^{-3}$, $Q = 1$, $\varepsilon = 0.5$ and $C_T = 0.5$. Fig. 3 illustrates the effects of the heat sink/source size on the streamlines, isotherms and the local entropy generation. As shown in this figure, the density of the nanofluid near the hot wall increases as the heat source and the heat sink are located opposite to each other, and the nanofluid therefore moves upward along the vertical wall. The density then decreases as the nanofluid reaches near the cold wall, leading to a downward movement along the vertical wall, so that a cell in the core of the cavity appears in the streamlines. With an increase of $B$, the velocity increases and the core cell becomes stronger. With increasing $B$, the temperature ($\theta$) has a larger increment and the buoyancy effect is enhanced as well. Therefore, enhancement of the right hand side of Eqs. (15) and (16) leads to increasing the horizontal and vertical velocities, as shown in Fig. 4a and b. As $B$ increases from 0.2 to 0.6, the nearly vertical shape of the isotherms in the middle of the cavity gets looser at the top and bottom regions. Considering its definition, it can be seen that the Nusselt number increases with a decrease of the temperature of the nanofluid at the beginning region of the heat source. The isothermal lines become nearly horizontal at $B = 0.8$ and near the source, and hence the temperature of the nanofluid increases at the beginning region of the heat source. At the ending region of the heat source, the minimum and maximum temperatures occur at $B = 0.2$ and $B = 0.8$, respectively, and, therefore, the maximum and minimum values of the Nusselt number are related to $B = 0.2$ and $B = 0.8$, respectively, (shown in Fig. 5).

Fig. 6 displays the effects of adding nanoparticles on the average Nusselt number in various ranges of $B$. The average Nusselt number decreases with an increase of the nanofluid volume fraction. The Nusselt number has its maximum value at $B = 0.2$, and its
minimum value at $B = 0.8$. The viscosity of the nanofluid enhances via increasing the nanofluid volume concentrations; in some papers it was mentioned that the effect of increasing the viscosity of the nanofluid on heat transfer is more than the effect of increasing the thermal conductivity at the same volume fraction, and, therefore, the heat transfer decreases with an addition of the nanoparticles especially for natural convection heat transfer.

In the case of natural convection in a porous medium, the role of the dynamic viscosity is more effective compared to that of a regular fluid. This is due to the fact that in a regular clear space, the interaction of the fluid and the solid boundaries, which results in high gradient velocity regions next to the walls, is limited to the solid walls boundaries. However, in a porous space, the interaction between the solid structure of the porous matrix and the nanofluid is within the entire porous space as well as the wall boundaries. Therefore, in the natural convection heat transfer mechanism occurring in the porous spaces, a reduction of heat transfer for smaller values of the Rayleigh number can be expected. The results of the very recent studies regarding the analysis of natural convection of nanofluids in a cavity filled with a porous medium, for instance the study of Ghalambaz et al. [33], also confirms the reduction of heat transfer in natural convection in porous media.

Fig. 6b shows the ratio of the Nusselt number of the nanofluid to that of the base fluid in which the minimum reduction rate of the Nusselt number belongs to $B = 0.2$, while its maximum decrease rate belongs to $B = 0.8$.

Fig. 7 indicates the variation of the total entropy generation with the nanofluid volume fraction. The entropy generation decreases with an increase of the volume fraction and a decrease of heat transfer irreversibility. It minimizes at $B = 0.2$ and maximizes at

![Fig. 13. Streamlines (a), Isothermal (b), and Local entropy generation (c) for Cu-water at $\phi = 0.005$, $Ha = 10$, $B = 0.5$, $Ra = 10^5$, $Q = 1$.](image)
Fig. 7b presents the entropy generation rate (the ratio of the nanofluid entropy generation of the nanofluid to that of the pure fluid). The minimum and maximum values of the entropy generation rate with the increase of the volume fraction are seen at $B = 0.8$ and $B = 0.2$, respectively. Fig. 8 indicates that the increase of the volume fraction for all values of $B$ accounts for the improvement of the thermal performance. The rate of thermal performance is defined as the ratio of the entropy generation rate to the rate of the thermal conductivity.
average Nusselt number. Better thermal performance of the system is observed as the rate of the thermal performance decreases. The optimum thermal performance then occurs at $B = 0.2$ and $\phi = 0.05$.

Fig. 9 presents the variations of the average Nusselt number with those of the Hartmann number. As shown in the figure, the average Nusselt number decreases as the Hartmann number increases. In general, an external magnetic field leads to suppression of the flow field, and, therefore, the average Nusselt number is expected to decrease with the Hartmann number.

Fig. 10 indicates the variation of the thermal performance rate with an increase in the Hartmann number. As is evident in the figure, the optimum thermal performance happens firstly at $B = 0.2$ and secondly at $B = 0.8$, so that the rate of thermal performance
decreases with an increase in the value of the Hartmann number. For the rest of the $B$ values, as the Hartmann number increases, the rate of the thermal performance either does not change obviously, or increases slightly.

As observed in Fig. 11, for all $\phi$ values, the average Nusselt number shows a sinusoidal-like relation with the variations of the angle, with the maximum value of the average Nusselt number occurring at $B = 0.2$ and at angles approximately close to 50 and 310°. As a result of the small sizes of the heat source and sink, the values of the average Nusselt number are near each other at the angles of 0 and 180°. But with an increase of $B$, the difference of the Nusselt numbers between the two angles of 0 and 180 is enhanced, so that it minimizes at an angle close to 180° and at $B = 0.8$. At $\Phi = 0$, the maximum and minimum values of the Nusselt number are seen at $B = 0.8$ and $B = 0.2$, respectively.

Fig. 12 presents the variation of the global entropy generation with the angle of cavity. The entropy generation includes two main parts: fluid friction irreversibility (FFI) and heat transfer irreversibility (HTI). An increase of the heat transfer then leads to an increase of the entropy. The reason is that the portion of HTI is bigger than that of FFI. For the points at which the Nusselt number has the maximum value, the generated entropy is also maximum and vice versa.

The streamlines are shown in Fig. 13a. By the movement of the source from 0.2 to 0.7, the cell core also moves from the left half to the right half near the center. The horizontal and vertical velocities versus $D$ can be seen in Fig. 14a and b. Fig. 13b displays the isothermal lines for different locations of the source and sink. As shown in this figure, the congestion of the isothermal lines shifts from the left wall at $D = 0.3$ to the right wall at $D = 0.7$. At $D = 0.4$ and $D = 0.6$, the isothermal lines have a relative symmetry at the sink. Fig. 13c shows the local entropy generation. As observed in the figure, the entropy generation rises from $D = 0.3$ to $D = 0.7$ at the source and at the right wall.

Fig. 15 presents the local Nusselt number along the heat source. Considering the isothermal lines, the maximum temperature occurs at $D = 0.3$ and at the beginning of the source in comparison with the other values of $D$. Hence the minimum value of the Nusselt occurs at $D = 0.3$. The temperature rises, and therefore the Nusselt decreases along the source, and it also rises slightly at the end of the heat source. As is evident in the figure, the local Nusselt number minimizes at the beginning of the source and at $D = 0.5$. Based on the definition of the average Nusselt number, the Nusselt number is supposed to maximize at $D = 0.5$ (as shown in Fig. 16a). Also, Fig. 16a indicates that the maximum and the minimum values of the average Nusselt number occur at $D = 0.5$ and $D = 0.3$, respectively. Furthermore, with an increase of the volume fraction, the average Nusselt number decreases for all values in the considered range of $D$. 

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Fig. 18. Variation of the average total entropy generation rate for Cu-water at $Ha = 10$, $B = 0.5$, $Ra = 10^3$, $Q = 1$, $\Phi = 15°$.

Fig. 19. Variation of global entropy to average Nusselt number at $Ha = 10$, $B = 0.5$, $Ra = 10^3$, $Q = 1$, $\Phi = 15°$. 
**Fig. 16b** shows the rate of the average Nusselt variations with the variations of the volume fraction of the nanofluid from $D = 0.3$ to $D = 0.7$. As shown in the figure, adding nanoparticles leads to a decrease of the heat transfer with the maximum value of reduction occurring at $D = 0.6$ and $D = 0.7$.

The variation of the total entropy generation with the nanofluid volume fraction against different values of $D$ is displayed in **Fig. 17**. By increasing the volume fraction of the nanofluids and considering the decrease of the heat transfer, and accordingly the heat transfer irreversibility, the generation of entropy decreases. The minimum and maximum values of the entropy generation are observed at $D = 0.7$ and $D = 0.3$, respectively.

**Fig. 18** shows the variation of the entropy generation rate with an increase of the volume fraction while $D$ ranges from 0.3 to 0.7. The maximum decrease of the rate of entropy happens at $D = 0.7$, whereas the minimum rate occurs at $D = 0.3$.

**Fig. 19** illustrates the variation of the thermal performance rate with the variation of the volume ratio against different values of $D$. As is evident in the figure, adding nanoparticles leads to an improvement of the thermal performance criteria. At $D = 07$, this improvement of performance is higher than that for the other values of $D$.

**Fig. 20a** shows the variation of the average Nusselt number with an increase of the Hartmann number. As the Hartmann number increases, the average Nusselt number reduces for different values of $D$. In general, an external magnetic field leads to suppression of the flow field, and, therefore, the average Nusselt number is expected to decrease with the Hartmann number. **Fig. 20b** shows the variation of the rate of the average Nusselt number based on the variation of the Hartmann number. As observed in the figure, the Nusselt number decreases with an increase of the Hartmann number. This decrease in the Nusselt number maximizes at $D = 0.7$.

**Fig. 21a** depicts the variation of the total entropy generation with the Hartmann number for values of $D$ ranging from 0.3 to 0.7. Via increasing the Hartmann number, the entropy generation decreases, because of the decrease in the heat transfer irreversibility and the Joule heat irreversibility. **Fig. 21b** shows the variation of the entropy generation rate against the Hartmann number. The entropy generation minimizes at $D = 0.7$ and $H = 25$.

**Fig. 22** shows the influence of the Hartmann number on the rate of thermal performance criteria for different values of $D$. As represented in the figure, an increase of the Hartmann number accounts for the improvement of the thermal performance at $D = 0.6$ and $D = 0.7$. Without a clear variation at $D = 0.5$, an increase of the Hartmann number leads to a decrease in the thermal
performance at \( D = 0.3 \) and \( D = 0.4 \).

Fig. 23 displays the variation of the average Nusselt number with the variation of the cavity angle. A sinusoidal-like behavior is also seen in this figure. For angles ranging from 0 to 360°, the maximum values of the average Nusselt number occur at \( D = 0.5 \) and the minimum values occur at \( D = 0.3 \) and \( D = 0.7 \). From the viewpoint of heat transfer, the most proper angles are 40–50 and 300–310° approximately, while the least proper angles are those ranging from 170 to 180°.
5. Conclusion

The effects of the sink and source location and size on the entropy generation and MHD natural convection flow in an inclined porous enclosure filled with a Cu-water nanofluid are investigated numerically. The results have led to the following concluding remarks:

1. The maximum value of the average Nusselt number occurs at $B = 0.2$ and at angles approximately close to 50 and 310°.
2. At $\Phi = 0$, the maximum and minimum values of the average Nusselt number are observed at $B = 0.8$ and $B = 0.2$, respectively.
3. Increasing the volume fraction of the nanoparticles decreases the convective heat transfer inside the porous cavity for all ranges of $B$ and $D$.
4. The average Nusselt number decreases considerably upon an enhancement of the Hartmann number for all the considered ranges of $B$ and $D$.
5. The increase of the Hartmann number accounts for the improvement of the thermal performance at $D = 0.6$ and $D = 0.7$.
6. From the view point of heat transfer, the most proper angles are 40–50 and 300–310° in the whole range of $D$.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cjph.2017.11.026.

References