Estimation of permeability of naturally fractured reservoirs by pressure transient analysis: An innovative reservoir characterization and flow simulation

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Abstract

Fluid flow processes in naturally fractured reservoirs are strongly controlled by fracture intensity, their orientation, and interconnectivity. Therefore, knowledge of fracture properties plays a critical role in reservoir management. Developing a detailed description of subsurface fracture map is challenging for geothermal and petroleum industry due to a number of reasons: It is often difficult to obtain sufficient hard data such as borehole images and core description. Also, there is a general lack of quality of interpretation of soft data, such as well logs, seismic attributes, and tectonic history etc. for fracture interpretation. To overcome the shortcomings the industry has been relying heavily on geo-statistical analysis of both hard and soft data.

In this paper we used well test data (dynamic data) to reduce the level of uncertainty of the existing methods for generating fracture map through an innovative inversion technique. The major inversion techniques include simulated annealing, sequential successive linear estimator, and gradient and streamline based method. In this paper we used gradient based method which utilises adjoint equation. The inversion is carried out in a number of steps. First we analyse hard and soft data to generate characteristic fracture properties such as fracture density and fractal dimension. Next we use geo-statistical technique to spatially distribute fractures. Then with use of gradient based technique, we optimise the fracture attributes through different realization. Next we use an innovative simulation of fluid flow through discrete fractures in 3D fractured porous media and estimated pressure and pressure derivatives. The pressure and pressure derivatives are compared with well test data taken from a typical fractured basement reservoir located in offshore Vietnam to determine percentage error.

The results showed that the simulated pressure change and pressure derivatives match well with well test data. This has allowed us to capture the complex subsurface fracture pattern. This vital information can help the operator design an effective reservoir management plan.

1. Introduction

Naturally fractured reservoirs host more than 50% of the world remaining hydrocarbon reserves (Gutmanis et al., 2015). Mechanism of fluid flow through such reservoirs is not well understood (Gang and Kelkar, 2006) and this is mainly because these reservoirs comprise of two mediums of diverse properties: rock matrix and fractures. Naturally fractured reservoirs have been classified according to the relative contribution of the matrix and fractures to the total fluid production (Nelson, 2001). Accordingly, the fractured reservoirs are classed as: Type 1 - fractures provide essential porosity and permeability, Type 2 - fractures provide essential permeability, and Type 3 - fractures provide permeability assistance. Fractures are mechanical breaks, which arise from stress concentrations around flaws, heterogeneities, and physical discontinuities. These characteristics properties affect fluid flow and mechanical stability of the reservoir. A comprehensive understanding of fluid flow mechanism in fracture-matrix system is an essential requirement for estimation of production potential of and selection of appropriate production methods for these reservoirs. Simulation of fluid flow for estimation of production potential of these reservoirs, however, is extremely challenging...
due to (a) complex fracture geometry (b) lack of information on fracture properties, which introduce a high level of uncertainty associated production estimation (Narr, 1996; Ortega et al., 2006).

The main objective of this paper is to present a methodology for characterizing the naturally fractured reservoirs in order to maximize the benefits of geomechanical control on predicting fracture properties, therefore intensity, geometry and orientation of fractures and their effect on fluid recovery. Characterization of fracture properties has been carried out by integrating information from two different sources; static data (seismic, well logs, core description, borehole images, tectonic history, geological structures etc.) and dynamic data (well test data and wells production history). Several methods have been used for generation subsurface fracture map based on static data. Most notable methods are Abdelazim and Rahman et al. (2014), Doonechaly and Rahman (2012), and Will et al. (2005). Among these, the paper by Doonechaly and Rahman (2012) describes a comprehensive methodology of generating subsurface fracture map. In this approach the two important parameters, fracture density and fractal dimension are estimated from a number of data sources including tectonic history reservoir structure, seismic attributes, well logs, borehole images and core description. Often it has been observed that all data sources, such as core description and borehole images are not always available for every well drilled in an area. Again cores and borehole images are taken from a section of the wellbore. In order to overcome discontinuity of the data in both horizontal and vertical directions a back propagation neural network is used to create a non-linear relationship of attributes of different data sources (for example, attributes of seismic, borehole sonic, Gama ray etc.) with the fractal dimension and fracture density. In the next step a nested neuro-stochastic simulation (sequential Gaussian approach) is used to generate subsurface fracture map based on fracture density and fractal dimension. In the final step simulated annealing is used to generate an optimised fracture map. This approach can be further enhanced integrating the fracture attributes from the outcrop if available. Most of the geological models of fractured reservoirs generated based on static data alone sometimes failed to reproduce wells production history (Ouenes et al., 2004; Efendiev et al., 2005). This is mainly because the hydrodynamic properties of fracture network are not considered in the statistical analysis.

Since 1970s, a significant progress has been achieved to generate a consistent methodology to characterize naturally fractured hydrocarbon and geothermal reservoirs by utilising well test and production data. This was achieved in two steps: First the static data is used to statistically generate subsurface fracture map with fracture intensity orientations and geometries in different realisations. Then inversion techniques are used to converse the simulated pressure data with that of well test data. These techniques include stochastic algorithms, gradient based and streamline based techniques (Chen et al., 1974, Chavent et al., 1975, Oliver, 1994, Landa et al., 2000, Zhang and Reynolds, 2002, Gang and Kelkar, 2006, Oliver and Chen, 2011; Azim, 2015).

Gradient based methods require an optimisation algorithm (e.g. Quasi-Newton, Conjugate Gradient, and Levenberg-Marquardt) and these algorithms need the derivative of the production responses with respect to the changes of the reservoir parameters. Chen et al. (1974) and Chavent et al. (1975) developed an efficient technique for derivatives calculation based on the adjoint equation (model) applied to single phase flow problems. Extension of these adjoint models for multiphase flow problems were introduced by Yang et al. (1988), Bissell (1994), and Zhang and Reynolds (2002). These methods tend to converge quickly, but subsurface heterogeneity mapping for fractured reservoirs by gradient base methods are simply inappropriate because they deal with block based permeability tensors. Stochastic simulation is another technique for generation of multiple realisations of the reservoir fracture property, rather than simply estimation of the mean value of the property. The most common stochastic methods applied for petroleum and geothermal engineering problems are simulated annealing (Gupta et al., 1994) and genetic algorithms (Goldberg and Holland, 1988; Carter and Ballester, 2004). In these methods, gradients are not required and instead evaluation of the forward simulation model is used. The disadvantage of these methods is that they require numerous simulation runs for convergence (Wu et al., 2002 and Liu and Oliver, 2004) which is considered computationally exhaustive for large-scale applications. Streamline based methods have been presented by Vasco et al. (1999) and Agarwal and Blunt (2003). These methods have significant advantages over simulated annealing due to two main reasons. First, it is extremely efficient in resolving complex heterogeneous reservoir because the solution methodology is faster than conventional methods. Second, the sensitivity coefficients are computed with only one single simulation run. The limitation of this method is that the model equations do not honour multipoint geostatistics.

Central to all inversion algorithms is the forward modelling/numerical simulation of fluid flow through matrix and fractures.
Currently, three major approaches are used to simulate fluid flow through naturally fractured reservoirs which include: continuum, dual porosity/dual permeability, and flow through discrete fracture approaches. In single continuum approach fractured medium is divided into a number of representative volumes and bulk macroscopic values of reservoir properties varying from point-to-point are averaged over the volume which is often known as blocked based permeability tensors (Lough et al., 1998; Park et al., 2002; Gupta et al., 2001; Sarkar et al., 2004; Teimoori et al., 2005; Fahad, 2013). Gupta et al. (2001) considered matrix with no permeability and simulated fluid flow through fractures only. Park et al. (2002), Gupta et al. (2001), Sudipata et al. (2004), and Teimoori et al. (2005) on the other hand present comprehensive methodologies for estimating permeability tensor for arbitrarily oriented and interconnected fracture systems.

In the dual continuum approach, the reservoir is divided into two major parts: fractures and matrix. According to Warren and Root (1963), fractured reservoirs are often visualized as a set of stacked sugar cubes (see Fig. 1b). In this approach, the fractures provide the main flow paths, while the matrix acts as a source of fluid. The fluid transfer between the fractures and the matrix is defined based on transfer functions. Duguid and Lee (1977), Pruess (1985), Gilman (1986) and Bourbiaux et al. (1999) introduced a range of different matrix/fracture transfer functions to simulate the fluid flow in large scales. Following this, a significant number of studies have been carried out both in analytical and numerical frameworks using dual porosity approach (Choi et al., 1997; Pride and Berryman, 2003; Gong et al., 2008).

Landereau et al. (2001) proposed a new technique for simulating flow in fractured porous media like rock masses. The proposed technique have been created to separate the contributions from the rock matrix and from the fracture systems (which are also treated as an equivalent continua), the so-called dual (double) porosity models, dual (double) permeability models and dual (double) continuum models, in order to simplify the complexity in the fracture–matrix interaction behaviour and to partially consider the size effects caused mainly by the fractures.

It has been shown that the advective transport component in more permeable matrix blocks (e.g., porous sandstones) results in additional solute transfer between fractures and matrix, which is not typically taken into account by dual-porosity concepts (Cortis and Birkholzer, 2008). Therefore, Noetinger and Estebenet (2000) proposed a new technique called continuous time random walk (CTRW) method. This method is an alternative to the classical dual-porosity models for modelling and upscaling flow and/or solute transport in fractured rock masses. Originally developed to describe electron hopping in heterogeneous physical systems (Scher and Lax, 1973).

One of the applications of classical dual porosity approach has been the analysis of pressure transient data from fractured porous media. The concept was presented by Warren and Root (1963). In this approach, the fluid flow from the matrix to the fractures is assumed to be pseudo steady state which means that the pressure at every point in the matrix elements decreases at the same rate.
(the reservoir is assumed to be homogeneous). Warren and Root (1963) developed an analytical model for unsteady state flow which allowed production analysts to analyse the pressure build up data in some details. In this analysis two parameters are used to describe the behaviour of the fractured porous media. Firstly the storativity ratio ($\omega$) was introduced to describe the fluid capacitance in fractures as:

$$\omega = \frac{\phi_2 c_2}{\phi_2 c_2 + \phi_1 c_1}$$

where $\phi_1$ is the matrix porosity, $\phi_2$ is the secondary porosity, $c_2$ is the total compressibility in the secondary system (fracture) and $c_1$ is the total compressibility in the primary system (matrix). The other parameter ($\lambda$) represents the interporosity flow coefficient related to degree of system heterogeneity and is expressed as:

$$\lambda = \frac{k_{\text{eff}} r_w^2}{k_2}$$

where $k_1$ is the matrix permeability, $k_2$ is the effective (average) permeability of the heterogeneous medium, $\alpha$ is the matrix shape factor and $r_w$ is the wellbore radius. This interporosity flow coefficient parameter is dimensionless and describes the ability of the fluid to flow from the matrix to the fracture network. Also, it gives an indication about how rapidly the matrix contributes to the

Fig. 4. Description of how the reservoir domain is discretised for hybrid methodology (a) the reservoir domain is divided into a number of grid blocks without considering of long fractures (b) 3D block based permeability tensor for short to medium fractures (c) the block based permeability tensors are distributed to the corresponding tetrahedral elements (matrix porous media) and long fractures are discretised explicitly.
production process. In order to simplify the solution, Warren and Root (1963) ignored the wellbore storage effects by assuming that the wellbore is a line source. Mavor and Ley (1979) extended the analytical solution of Warren and Root (1963) by including the wellbore storage effects by incorporating a finite wellbore radius. Bourdet et al. (1983) used Mavor and Ley’s (1979) analytical model

![Fig. 5. Generated fractures for the studied reservoir (the plot shows that most of the fractures direction toward x- axis).](image)

![Fig. 6. Polar diagram for directional permeability: a) permeability tensor for $k_{xx-yy}$, b) permeability tensor for $k_{xx-zz}$, and c) permeability tensor for $k_{yy-zz}$.](image)

<table>
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<tr>
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</thead>
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<tr>
<td>Reservoir dimensions</td>
<td>100 m x 100 m x 200 m</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.01 mD</td>
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<tr>
<td>Matrix porosity</td>
<td>0.002</td>
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<tr>
<td>Initial reservoir pressure</td>
<td>34.9 MPa (5,063 psia)</td>
</tr>
<tr>
<td>Injection pressure (injection case)</td>
<td>54.9 MPa (7963.65 psia)</td>
</tr>
<tr>
<td>Fracture properties</td>
<td></td>
</tr>
<tr>
<td>Fracture aperture</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>Fracture Intensity</td>
<td>0.12 (m$^3$/m$^2$)</td>
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<tr>
<td>Fractal dimension, D</td>
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</tr>
<tr>
<td>Fluid properties</td>
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<tr>
<td>Viscosity</td>
<td>1.38 cP</td>
</tr>
<tr>
<td>Compressibility</td>
<td>1.0E-06 psi$^{-1}$</td>
</tr>
<tr>
<td>Density</td>
<td>1000 kg/m$^3$</td>
</tr>
</tbody>
</table>

Table 1

Reservoir inputs parameters for validation of permeability tensor calculations.
and presented the first pressure derivative plot for analysing pseudo steady state flow transfer in a dual porosity system.

Kazemi (1969) presented a numerical approach, as an alternative to Warren and Root’s (1963) model. Kazemi (1969) studied the transient flow condition between the matrix and fractures by representing the reservoir as an equivalent system of horizontal fractures embedded between matrix layers. Bourdet and Gringarten (1980) and Bourdet et al. (1984) presented an analytical solution for the Kazemi’s (1969) transient model to include the wellbore storage effects and skin. Following the work of Bourdet et al. (1984) and Braester (1984) studied the effect of the matrix block size on the pressure derivative for transient flow and observed that the matrix block size has no effect on the behaviour of the pressure change and its derivatives.

The main drawbacks of the aforementioned approaches are: (1) the fluid distribution within the matrix blocks remains constant during the simulation period, (2) the model can be applied to only interconnected fractures and is applicable to a small number of large scale fractures in flow simulation.

In discrete fracture approach, each fracture surface is represented by individual attributes of permeability and aperture. The size, spatial location, orientation and intensity of these surfaces have a substantial importance and are accounted for during the generation of discrete fracture network. The fluid flow equations through the fracture network and matrix system are solved using both analytical and numerical methods, e.g. finite element method (Karimi-Fard et al., 2004); finite volume method (Reichenberger et al., 2006), mixed finite element and finite volume method (Geiger et al., 2004). The limitation of this approach is that an exhaustive computation time is required which prevents its application to simulation of fluid flow in large number of fractures. Doonechaly and Sheik (2013) extended the work of Teimoori et al. (2005) by introducing hybrid of single continuum and flow through discrete fractures (fractures that are greater than the set threshold value. This has allowed the authors to simulate flow through hundreds of long fractures that are greater than 100 m (Abdelazim and Rahman et al., 2014).

Noetinger (2014) and Noetinger (2015) they simulated a single-phase fluid flow inside a fractured medium. The authors consider the rock surrounding the fractures as impervious and model the flow by Darcy’s law. The medium is approximated by a Discrete Fracture Network (DFN) in which the fractures are assumed to have a negligible thickness with respect to the other dimensions, and are represented as planar polygons intersecting each other in three dimensional space, with an equivalent bi-dimensional conductivity obtained by averaging the tri-dimensional one along the negligible dimension.

Wei et al. (1998) developed a 3D numerical model in order to simulate pressure transient through fracture/matrix using a dual porosity model. This simulation allows the determination of fracture transmissivities and the quantification of fracture pore volume. The authors showed that dual porosity model failed to describe the behaviour of fluid flow through fractured system in many cases. Carlson (2003) was using specific transfer functions to simulate the flow transfer from fractures to matrix. It was assumed that the fractures provide the main flow conduit and matrix acts as a source/sink to the fractures. Kamal et al. (2005) reviewed the limitation of traditional well testing, and presented a new method called Numerical Well Testing to incorporate the wealth information contained in the transient behaviour of the tests. In this study, reservoir geological factors (as reservoir heterogeneities, complicated boundary, multi-phase flow and production history) are considered. This method is applied for conventional reservoirs (not fractured reservoirs) using different field cases to match the well test data and field production history. Basquet et al. (2005) used a homogenization method (note: we need to explain homogenization in the light of flow simulation). The homogenization method used in different steps to simulate pressure transient through fractured system. The first step is subdividing the fractured reservoir into two scales: local scale and global scale. The local scale is for a single matrix surrounded by fractures and large scale is the grid block length. In the next step of homogenization process, the flow equations (mass balance and Darcy’s equation) are described on the local scale (matrix). Then, the differentiation operator in the flow equations is split into a global and local scale terms for solving the flow equations numerically. The splitting procedure is only apply for fracture equation.

Will et al. (2005) presented a method to integrate seismic anisotropy attributes with reservoir-performance data (well test and production data) for characterization of naturally fractured reservoir through the use of block based permeability tensors and Eclipse 100 as a forward model.

Recently, there are many studies revolving the use of pressure transient data for characterising naturally fractured reservoir through inversion of well test data. Morton (2012) presented two new techniques used to calibrate with the inversion of well-test data by integrating a reservoir flow simulation model and an inversion technique. In this approach a boundary element method is used to simulate fluid flow and study the pressure transient behaviour of the reservoir with arbitrary distributed vertical fractures. Kuchuk and Biryukov (2012) presented an analytical solution and thus can be easily extended to incorporate other reservoir features such as sealing or leaky faults, domains with altered petro-physical properties in order to understand pressure behaviour of continuously and discretely fractured reservoirs. The author showed that Warren and Root’s (1963) dual-porosity model is not adequate for pressure transient well-test interpretations and the pressure behaviour of discretely fractured reservoirs shows many different flow regimes depending on fracture distribution, its intensity and conductivity.

In this paper, pressure transient data from a fractured basement reservoir offshore Vietnam has been used to evaluate the fracture map which is generated by statistical analysis of field data (static data) as per Doonechaly and Rahman (2012). This is carried out in two inversion steps. In the first inversion step, the reservoir is divided into a number of grid blocks and the block based permeability tensors are estimated by considering all fractures (small to large fractures) that are intersected by the blocks. Fluid flow is simulated (forward modelling by single continuum approach, therefore the permeability tensors) to estimate change in pressure and pressure derivatives. The simulated pressure data is compared with that obtained from well test to evaluate error. The gradient based technique is utilised to repeat the forward modelling for different realisations of block based fracture permeability (permeability tensors) until the error is reduced to zero.

### Table 2
Reservoir inputs data for a typical fractured basement.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir dimensions</td>
<td>500 m × 500 m × 250 m</td>
</tr>
<tr>
<td>Vertical well is partially</td>
<td>penetrated the formation</td>
</tr>
<tr>
<td>thickness (90 m)</td>
<td></td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.0095 md</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>25</td>
</tr>
<tr>
<td>Fracture aperture</td>
<td>7.96 × 10⁻³ mm</td>
</tr>
<tr>
<td>Initial fracture intensity</td>
<td>0.15 m⁻¹</td>
</tr>
<tr>
<td>Initial reservoir pressure</td>
<td>34.9 MPa (5063 psia)</td>
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<tr>
<td>Fluid viscosity</td>
<td>1.38 cp</td>
</tr>
<tr>
<td>Fluid compressibility</td>
<td>10⁻⁸ MPa⁻¹</td>
</tr>
<tr>
<td>Production time before</td>
<td>72 h</td>
</tr>
<tr>
<td>shut in (tₚ)</td>
<td></td>
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Wei et al. (1998) developed a 3D numerical model in order to simulate pressure transient through fracture/matrix using a dual porosity model. This simulation allows the determination of fracture transmissivities and the quantification of fracture pore volume. The authors showed that dual porosity model failed to describe the behaviour of fluid flow through fractured system in many cases. Carlson (2003) was using specific transfer functions to simulate the flow transfer from fractures to matrix. It was assumed that the fractures provide the main flow conduit and matrix acts as a source/sink to the fractures. Kamal et al. (2005) reviewed the limitation of traditional well testing, and presented a new method called Numerical Well Testing to incorporate the wealth information contained in the transient behaviour of the tests. In this study, reservoir geological factors (as reservoir heterogeneities, complicated boundary, multi-phase flow and production history) are considered. This method is applied for conventional reservoirs (not fractured reservoirs) using different field cases to match the well test data and field production history. Basquet et al. (2005) used a homogenization method (note: we need to explain homogenization in the light of flow simulation). The homogenization method used in different steps to simulate pressure transient through fractured system. The first step is subdividing the fractured reservoir into two scales: local scale and global scale. The local scale is for a single matrix surrounded by fractures and large scale is the grid block length. In the next step of homogenization process, the flow equations (mass balance and Darcy’s equation) are described on the local scale (matrix). Then, the differentiation operator in the flow equations is split into a global and local scale terms for solving the flow equations numerically. The splitting procedure is only apply for fracture equation.

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minimum. The optimised permeability tensors are then correlated to fracture properties of the corresponding blocks. In the next inversion step, different subsurface fracture maps are realised based on the correlation and the forward modelling carried out by using single continuum and discrete fracture approach, which was developed by Gholizadeh and Abdelazim et al. (2015) until an optimised fracture map is obtained.

1.1. Fracture data analysis

Fracture data analysis is the first step in reservoir characterization process. The analysis consist of the determination of types of fractures or fracture parameters that control the distribution and quality of flow zones. Borehole images and production data are used to identify a set of variables such as dip, azimuth, aperture, or density that controlling hydrocarbon flow. Fracture indicators such as production rates are combined with borehole images to flag the flow contributing fracture zones. This technique has been used successfully in fractured basement reservoirs (Tandom et al., 1999; Luthi, 2005). The fracture sets are defined based on fractures dip, length, and azimuth. The initial data of fractures length and dip angles ranging from 9 m to 60 m and 70° to 90° respectively and the fracture aperture ranges from 0.004 mm to 0.04 mm. Once the fracture set has been identified, it is used in the form of a fracture intensity curve. Generation of intensity logs from fracture data is essential in the data analysis.

Fig. 7. Schematic representation of reservoir pressure (Top) after (a) 1 year (left) and (b) 10 years (right) of water injection and fluid velocity (bottom) after (c) 1 year (left) and (d) 10 years (right) of water injection with $P_m = 54.9$ MPa, $\Delta p = 41.14$ MPa, and $p_i = 34.6$ MPa. Velocity contour map in (m/s) and pressure contour map in (psi).
Fracture intensity. Fig. 2 shows a diagram for dip angle and radius of the generated fractures for the first fracture realization. The fractures are generated stochastically using Gaussian stochastic simulation in which each fracture feature is generated based on the random realization and it continues until the total fracture intensity and fractal dimension of the studied area are met. The subsurface fracture map of the area (which include top, middle and bottom zone) around the tested well is generated by using the calculated fracture intensity of 0.4 m$^{-1}$. Fig. 3 presents initial realization of the generated subsurface fracture map.

1.2. Simulation of fluid flow in fractured reservoirs

In this paper, a hybrid methodology, which combines the single continuum and the discrete fracture approach in a poroelastic framework, is utilised to simulate fluid flow. A typical fractured basement reservoir is used for flow simulation. In the proposed methodology small to medium fractures (fractures $1 < 40$ m, 4500 fractures) along with their original properties (orientations and locations) are considered as part of the matrix (in the form of permeability tensor, which can be contributing to local vertical heterogeneity) and long fractures (1 $\geq$ 40 m, 2000 fractures) along with their original properties (orientations and locations) are considered as discrete fractures. For this purpose a threshold value for fracture length is defined. Fractures with the length smaller than the threshold value are used to generate the grid based permeability tensor (3D). Fractures with length longer than the threshold value, are explicitly discretised in the domain by using the tetrahedral elements. Single phase flow is then simulated by coupling permeability tensors and flow through discrete fractures in a poroelastic framework. The threshold length has been selected depends on the effect of different fracture length on the reservoir performance. A preliminary run was performed to predict fluid recovery from the studied reservoir using all fractures as a discrete fractures (using our novel multiphase fluid flow simulator in fractured reservoirs, Azim, 2015). Then we started to remove short fractures in length and equivalent permeability tensors have been used with trying to match the oil recovery with the first run. This loop is repeated by removing short fractures in length until a poor match occurred (between oil recovery resulted from the first run and oil recovery by using equivalent permeability tensors). This poor matching gave a clue about the minimum fracture threshold length in which we have to maintain to preserve the matching process.

1.3. Simulation of block based permeability tensor

First, the reservoir is divided into a number of grid blocks (1875 grid blocks) with dimension of (20 m $\times$ 20 m $\times$ 30 m) (see Fig. 4a) and short to medium fractures, that cut these blocks, are used to calculate permeability tensors (see Fig. 4b). The grid block (domain) along with small and medium fractures as defined by the threshold value is discretised using tetrahedral elements in 3D domain for matrix and by triangular elements in 2D domain for fractures. Once the block based permeability tensors (3D) are calculated the reservoir domain along with long fractures are discretisation by tetrahedral elements for matrix as well as triangular elements for fractures as shown in Fig. 4c. The used grid blocks were built by using in house developed mesh generator and the grid block size was selected arbitrarily to be less than the minimum discrete fracture length (40 m) to honour the reservoir heterogeneity. The used method for calculating permeability tensor in this paper has been validated in Azim (2015).

In this paper, the grid based permeability tensor includes diagonal and off diagonal terms which are calculated by using three dimensional flow equations given by Darcy’s law and continuity equation. In
this method, pressure and flux boundary conditions that have been presented by Durlofsky (1991) are used along the block boundaries. The major drawback of this method is that it assumes no flux interchange between matrix and fractures. Nevertheless, this approach is useful for rocks with negligible permeability or where the matrix contribution is known to be very small (like in fractured basement reservoirs). A well comparison has been made by Anupam (2010) to compare the effective permeabilities obtained from the percolation analysis to the actual flow based upscaled permeability in case of the matrix contribution is very low. The comparison results show a good agreement between the two methods for computing effective permeability, and this confirm that we can use the flow based upscaled permeability along with Durlofsky (1991) periodic boundary conditions in our study. The governing flow equations and their discretisation in a finite element formulation are presented in Appendix A.

1.4. Validation of permeability tensor methodology

The main aim of this part is to calculate the permeability tensor for the generated subsurface map in Fig. 5 and 6 using Oda (1980) method and our developed numerical simulator as well to check
its reliability. Input reservoir parameters that have been used during the calculations process are provided in Table 1. The fracture aperture is assumed to be 0.1 mm with fracture radius ranging from 40 m to 100 m. The number of fractures in the entire reservoir region is about 2000 fractures. The permeability tensor is calculated by considering the reservoir is one block and the results of the numerical simulator are compared against the Oda’s analytical solution.

Using the analytical equations derived by Oda (1985), the calculated permeability tensors are as follow:

\[
\begin{bmatrix}
    k_{xx} & k_{xy} & k_{xz} \\
    k_{yx} & k_{yy} & k_{yz} \\
    k_{zx} & k_{zy} & k_{zz}
\end{bmatrix} =
\begin{bmatrix}
    4.246 & 0.6418 & 0.0167 \\
    0.6418 & 3.2708 & 0.00817 \\
    0.0167 & 0.00817 & 7.3243
\end{bmatrix}
\] (3)

The results of permeability tensor calculations using the developed numerical simulator are as follow:

![Fig. 12. Plot of fracture intensity versus mean square permeability.](image1)

![Fig. 13. (a) 3D optimised fracture map generated using the optimised block based permeability tensors and (b) initial 3D fracture map generated using object based model.](image2)

![Fig. 14. Measured and simulated shut in pressure after inversion at wellbore location with using the optimised subsurface fracture map presented in Fig. 3, fractal dimension = 1.25, matrix permeability = 0.0095 md, P_{initial} = 5063 psi and production time before shut in = 72 h. This plot is produced based on hybrid approach.](image3)

![Fig. 15. Pressure change and pressure derivatives after inversion at wellbore location with using the optimised subsurface fracture map presented in Fig. 13, fractal dimension = 1.25, matrix permeability = 0.0095 md, P_{initial} = 5063 psi and production time before shut in = 72 h. This plot is produced based on hybrid approach.](image4)
The permeability tensors values from Oda’s analytical and numerical solutions are in a good agreement.

\[
\begin{bmatrix}
k_{xx} & k_{xy} & k_{xz} \\
k_{yx} & k_{yy} & k_{yz} \\
k_{zx} & k_{zy} & k_{zz}
\end{bmatrix} = \begin{bmatrix}
4.195 & 0.7218 & 0.0235 \\
0.7218 & 3.458 & 0.00625 \\
0.0235 & 0.00625 & 7.135
\end{bmatrix}
\] (4)

The permeability tensors values from Oda’s analytical and numerical solutions are in a good agreement.

1.5. The hybrid approach

Single phase fluid flow in a typical fractured basement reservoir is simulated by coupling 3D permeability tensors with flow through discrete fractures. Long fractures ($l \geq 40$ m) along with their original properties (orientations and locations) are discretised explicitly within the reservoir domain. The reservoir and fluid properties used in fluid flow simulation are presented in Table 2. Results of velocity and pressure fields for the case of injection are presented in Fig. 7. Change in reservoir pressure and fluid velocity due to water injection is shown in Fig. 7(a), (b), (c), and (d) after 1 and 10 years respectively. As can be seen from these figures, the permeability of the discrete fractures has a significant effect on pressure diffusion. Fluid is moving fast through the interconnected fractures and after 10 years most of the reservoir is swept by injected fluid. The reservoir pressure and velocity is significantly high around the injection well and with continuous fluid circulation for 10 years, pressure and fluid velocity reach to a
quasi-steady state and the variation of reservoir pressure and fluid velocity across the reservoir remain roughly constant.

1.6. Inversion of well test data

In this section, well test data (dynamic data) are used to reduce the level of uncertainty associated with generation of subsurface fracture map by statistical analysis of field data (static data). In the context of inversion of well test data flow simulation by hybrid approach (forward simulation) plays a critical part as it traces the flow path and consequent pressure changes along the fractures, thus allowing interpretation of well tests data for characterising fracture properties. In view of this the simulated pressure changes for a typical fractured basement reservoir from Vietnam offshore is compared with that of the well test data (see Fig. 8). The results show that although the pressure change in both cases shows a similar trend there is a large discrepancy between the simulated pressure data and that of the well test data. This discrepancy is mainly due to the fracture map generated based statistical analysis field data which has failed to mimic well production history. There are numerous techniques available in the literature as discussed in the introduction section which can be used to reduce the uncertainty associated with the statistical analysis. In this paper, an innovative gradient based inversion of well test data is utilised. Few papers are focused on history matching process in fractured reservoirs by using optimisation technique. Among these, studies by Suzuki et al. (2005), Cui and Kelkar (2005) and Gang and Kelkar, (2006) are noteworthy. In these papers the authors used a gradient based optimisation technique to reduce the error between the estimated pressure data and the well test data. In their optimisation approach the authors used dual porosity approach and Eclipse 100 to perform the forward numerical flow simulation. Gang and Kelkar (2006) proposed adjusting the fracture permeability instead of the grid block effective permeability during the history matching. Other approaches have used fracture intensity as a history matching parameter (Suzuki et al., 2005; Cui and Kelkar, 2005), where the effective permeability of a grid block was computed from the fracture intensity A similar method in which cell properties were computed based on the number of fractures that cross each cell was presented by Hu and Jenni (2005). In their work, they proposed a generalization of the gradual deformation method for history matching object-based models. The method was applied to a fracture network, where fractures were modelled using an object-based approach (Cacas et al., 2001). The history matching was conducted by moving the fracture location along trajectories defined by an algorithm for migrating general Poisson point patterns. Dual-media approaches have also been used with the gradual deformation technique for history matching fractured reservoirs (Jenni et al., 2004). While Verscheure et al. (2012) proposed a new methodology to perform the history matching of a fractured reservoir model by adapting the sub-seismic fault properties and positions.

The scale description of the gradient based optimisation technique is presented in Appendix B. The main purpose of this inversion technique is to change a parameter (such as the permeability) until the objective function is achieved, that is the difference between the simulated and field observed data (pressure) is minimised. The gradient based optimisation technique is carried out in three steps: (a) fluid flow simulation by the forward numerical model, (b) adjoint system formulation, and (c) the sensitivity coefficient formulation. The derivation of forward numerical model along with adjoint system and the sensitivity coefficient formulations are described in details in Appendix B.

2. Results

In Fig. 9, a flow chart describing different steps used to optimise the subsurface fracture map is presented. The estimated block based permeability tensors (which include all fractures, mall to large fractures) for the subsurface fracture map (as presented in Fig. 5) are presented in Fig. 10. After the initial inversion (first step inversion), the optimised block based permeability tensors of the reservoir are presented in Fig. 11. A convergence criterion is applied in the first inversion step to achieve minimum error between the simulated and measured well test data. It is evident from the results that the region around the wellbore is mostly affected by the permeability changes during the first step of inversion. This is because, the pressure build up test is carried out on a single wellbore and as a consequence of this pressure changes only take place around the wellbore. It can also be observed from these figures that the change in permeability is directional. This means that the fracture flow properties, such as the fracture geometry, aperture, connectivity and orientation have an effect not only on the change in magnitude of permeability also the direction of permeability.

In the second inversion step, a relationship between the fracture intensity and the optimised permeability for each block is established. Fig. 12 shows the fracture intensity versus block based permeability tensor. Based on the plot (see Fig. 12), Eq. (5) for best fit is obtained (see below):

\[ FI = 0.0104 \sqrt{KRMS} - 0.002 \]

where FI is the fracture intensity and \( KRMS \) is the optimised root mean squared permeability of the block. Eq. (5) gives block based fracture intensity which is then used to generate different realisations of subsurface fracture map. The forward numerical modelling with the Hybrid approach is used in the 3rd inversion step to simulate fluid flow through different subsurface fracture networks. The simulated change in pressure and pressure derivatives are compared with that from the well test data for each realization until the error is minimised. The optimised discrete fracture map of the reservoir with minimum error is presented in Fig. 13. The simulated and measured pressure, change in pressure and pressure derivatives for the optimised subsurface fracture map are presented in Figs. 14 and 15 respectively. In Fig. 15, the build-up test data presented by Farag et al. (2010) are juxtaposed with that produced in this study. From the plot it can be seen that the optimised data produced in this study and that by Farag et al. (2010) match quite well with the well test data. Although the results of pressure data from both studies show a good match with the well test data, it is interesting to note that it is possible to show changes in permeability (directional), pressure and velocity profile near the wellbore region during the process of optimisation (as presented as part of this study). This means that the optimisation process not only changes the permeability, also it changes fracture properties, therefore the fractures orientation and distribution around the tested well.

3. Discussion

From the pressure derivative plot (see Fig. 15) three distinct flow regimes can be identified: the early time, the mid time and late time flow regimes. In the early time, flow regime is a spherical marked by a negative slope (negative slope of the pressure derivative curve) which is due to partial penetration of the reservoir (90 m open hole out of 250 m) by the producer. Due to the partial penetration, flow to wellbore changes from radial to spherical at
the base of the well. According to Brongs and Marting (1961) and Oddeh and Babu (1990) the deviation from radial to spherical flow is due to restricted fluid entry to the wellbore which contributes to an additional pressure drop near the wellbore and can be interpreted as skin factor. This flow regime occurred due to using of partially penetrated well (90 m out of 250 m, total formation thickness). The mid time flow regime is a short radial flow marked in the pressure derivative curve as a flat trend. Noteworthy is that both early and mid-time flow regimes behave in the same way as that of a porous media. This can be explained by the fact that the well is hardly penetrated by any fracture or medium to high fracture intensity region (see Figs. 16 and 17). The late time flow regime is a linear flow as recognized by a positive half-slope of the derivative curve which is caused by the fluid flow in discrete fractures. The geometry of linear flow regime consists of strictly parallel flow vectors. The parameters that can be calculated using this flow regime are the permeability of the formation in the direction of the flow vectors and the flow area normal to the flow vectors. In case of the fractured reservoir, the slope of the straight line fitting the data on the derivative plot can be used to determine the fracture length (Bourdet, 2002).

4. Conclusion

In this paper, the governing equations of single phase fluid flow are derived from mass and momentum balance equations. These equations are used to simulate fluid flow in discrete fractures and porous matrix medium. The single phase fluid flow is used as a forward fluid flow model in history matching of well test data by gradient based optimisation method. Pressure change and pressure derivatives for different fracture patterns, which are generated in different realisations using an object based model, are estimated and compared with the well test data. This process is continued until the difference between the simulated pressure data and well test data is reduced to minimum.

The results show that, the simulated well test data from the first realization of fractures vary significantly (> 2%) from the actual data. The inversion process has allowed us to modify the simulated pressure data by changing the fracture parameters through different realizations which in turn changed the flow properties of fracture. This means that, had the 3D simulation of fluid flow in discrete fractures along with the inversion technique not been used to match the pressure transient data, it would not have been possible to generate a realistic image of the subsurface fracture map, which includes fracture distribution, orientation, and connectivity between fractures. In addition, the workflow presented in this study can effectively traces fluid flow in fractures around the wellbore and overcome the limitations of conventional well test analysis presented by Farag et al. (2010). Thus, in a fractured reservoir well test data serves as critical information to evaluate fracture properties near wellbore region. It is, therefore, suggested to have multiple well test data (interference well test) which can allow field wide optimisation of fracture properties.

Appendix A: Calculations of 3D permeability tensors

Three-dimensional flow equations used for permeability-tensor calculations are given by Darcy’s law and continuity equations as follow:

\[
\begin{align*}
\text{v} &= \left( \frac{k}{\mu} \right) \text{∇p} \\
\n\end{align*}
\]

where \( v \) is the fluid velocity vector, \( \frac{k}{\mu} \) is the permeability tensor and \( \text{∇p} \) is the local pressure gradient. The grid based permeability tensors for short to medium fractures are calculated by using finite element technique. Each fracture is represented as a sandwiched element (triangular element) between three dimensional tetrahedral elements representing the matrix porous media as shown in Fig. A.1. Single phase fluid flow governing equation through matrix porous medium can be written as follows:

\[
\begin{align*}
\text{∇} \cdot \left( \frac{k}{\mu} \text{∇p} \right) + Q_{\text{fl}} &= 0 \\
\text{∇} \cdot \text{v} &= 0
\end{align*}
\]

where \( k \) is a full permeability tensor, \( \mu \) is the fluid viscosity, \( Q_{\text{fl}} \) is fluid source/sink term represents the fluid exchange between matrix and fracture, or fluid extraction (injection) from the wellbore.

Single phase fluid flow governing equation through discrete fractures can be expresses as:

\[
\begin{align*}
\text{∇} \cdot \left( \frac{k_f}{\mu} \text{∇p} \right) + q^+ + q^- &= 0
\end{align*}
\]

where \( q^+ \) and \( q^- \) are the leakage fluxes across the boundary interfaces, \( \text{∇} \) is the divergence operator in local coordinates system (Watanabe et al., 2010), and \( P_f \) is the pressure inside the fracture. The permeability of the fracture can be expressed by a parallel plate concept (cubic law) as shown in Eq. (A.6). It has been assumed that the fracture surfaces are parallel and the fluid-flow through a single discrete fracture is laminar (Snow, 1969).

\[
\begin{align*}
k_f &= \frac{b^2}{12}
\end{align*}
\]

where \( b \) is the fracture aperture, and \( k_f \) is the fracture permeability. As a result of superimposing fracture/matrix system, the fracture/matrix interface is directly treated by using the finite element technique (see Fig. A.1).

Finite element method formulation

The weighted-residual method is used to derive the weak formulation of the governing equation of fluid flow through a fractured system. Standard Galerkin method is applied to discretise the weak forms with appropriate boundary conditions (Zimmerman and Bodvarsson, 2000).

Eqs. (A.3) and (A.5) are written separately for matrix porous medium and discrete fractures in finite element formulation. The matrix is discretised using 3D tetrahedral elements and fractures are discretised by using 2D triangle elements. If FEQ represents the flow equation, the integral form can be written as follow:
\[
\int \int \int \int \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega = + \bar{\Omega} = \int \int \int \int \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega 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\[
\overrightarrow{M} = \int_{\Omega} \nabla N_p \frac{\nabla}{\mu} \nabla N_p^T \, d\Omega + \int_{\Gamma} \nabla N_p^T \frac{\partial}{\mu} \nabla N_p^T \, d\Gamma 
\]
(A.14)

\[
\overrightarrow{f} = -\int_{\Omega} N_p^T Q_p \, d\Gamma - \overrightarrow{M_p} \times \overrightarrow{p} - \overrightarrow{M_m} \times \overrightarrow{p}^{m-1}
\]
(A.15)

where \( m \) is referring to matrix porous medium and \( f \) to fracture network.

**Boundary conditions**

Pressure and flux boundary conditions that have been presented by Durlofsky (1991) are used for calculations of permeability tensor in fractured porous medium. These conditions are listed in Eq. (A.16) to Eq. (A.21) and Fig. A.2.

\[
p(y, x = 0, z) = p(y, x = 1, z) = G \text{ on } \partial D \text{ and } \partial D_q \\
u(y, x = 0, z) \cdot \overrightarrow{n}_1 = -u(y, x = 1, z) \cdot \overrightarrow{n}_4 \text{ on } \partial D \text{ and } \partial D_q \\
p(y = 0, x, z) = p(y = 1, x, z) = G \text{ on } \partial D \text{ and } \partial D_q \\
u(y = 0, x, z) \cdot \overrightarrow{n}_1 = -u(y, x = 1, z) \cdot \overrightarrow{n}_4 \text{ on } \partial D \text{ and } \partial D_q \\
p(y, x, z = 0) = p(y, x, z = 1) = G \text{ on } \partial D \text{ and } \partial D_q \\
u(y, x, z = 0) \cdot \overrightarrow{n}_5 = -u(y, x = 1, z) \cdot \overrightarrow{n}_5 \text{ on } \partial D \text{ and } \partial D_q \\
\]
(A.16) to (A.21)

where \( \mathbf{n} \) is the outward normal vector at the boundaries, \( \partial \mathbf{D} \) is the grid block boundary and \( \mathbf{G} \) is the pressure gradient. Specifying a zero pressure gradient along the y-, z- faces (\( \frac{\partial}{\partial y} = 0 \) and \( \frac{\partial}{\partial z} = 0 \)) and solving Eq. (A.12) with using the abovementioned periodic boundary conditions, the average velocity along x-, y- and z- directions can be calculated as follow:

\[
\langle \overrightarrow{v}_1 \rangle = -\int_{\partial \Omega} v \cdot \overrightarrow{n}_3 \, d\Omega \\
\langle \overrightarrow{v}_2 \rangle = -\int_{\partial \Omega} v \cdot \overrightarrow{n}_4 \, d\Omega \\
\langle \overrightarrow{v}_3 \rangle = -\int_{\partial \Omega} v \cdot \overrightarrow{n}_5 \, d\Omega \\
\]
(B.1) to (B.22)

\[
\phi^k = \frac{\partial c_i}{\partial t}
\]
(B.3)

where

\[
R_{ij,k} = \frac{\phi c_i}{\Delta t}
\]
(B.4)

\[
F_m^i(p^{i+1}, k_x, k_y, k_z) = A_m^{i+1} - A_m^i
\]
(B.5)

\[
A_m^i(p^i, \phi) = R(p^i+1)
\]
(B.6)

**Appendix B: Derivations of gradient based method equations**

**Adjoint system formulation**

To formulate Adjoint system of equations, the single phase flow equation is discretised in the following form:

\[
\nabla \cdot (k \nabla \phi - \mathbf{G}(x, y, z)) - q(x, y, z, t) = \phi c_i \frac{\partial p}{\partial t}
\]
(B.1)

\[
F_m^i(p^{i+1}, k_x, k_y, k_z) = R_{ij,k}(p_{ij,k}^{i+1} - p_{ij,k}^i)
\]
(B.2)

\[
\]

where

\[
R_{ij,k} = \frac{\phi c_i}{\Delta t}
\]
(B.3)

\[
F_m^i(p^{i+1}, k_x, k_y, k_z) = A_m^{i+1} - A_m^i
\]
(B.4)

\[
A_m^i(p^i, \phi) = R(p^i+1)
\]
(B.5)

**Travel time shift**

Three different ways used to represent the production data misfit, namely, the “amplitude misfit”, “travel time misfit”, and “generalized travel time misfit”.

The “travel time misfit” is defined as where the misfit is obtained
by lining up the observed and the predicted data at a reference time such as the breakthrough or the first arrival time. The travel time misfit has major advantages compared to the amplitude during inversion, first it has a quasi-linear properties compared to the high non linearity of the amplitude as a result the travel time inversion is robust and converge rapidly even if the initial model is far away from the solution. Second, it is computationally efficient because the number of travel-time is equal to the number of wells, regardless of the number of data points. This leads to considerable savings in computational time during the minimization (Cheng et al., 2003).

$$V_{kx}[L_{m}^{i+1}] = V_{ky}[L_{m}^{i+1}] = V_{kz}[L_{m}^{i+1}] = 0 \quad \text{(B.7)}$$

$$V_{kx}[L_{m}^{i}] = V_{ky}[L_{m}^{i}] = V_{kz}[L_{m}^{i}] = 0 \quad \text{(B.8)}$$

Now, the generalized travel time shift at each well can be denoted by $g(i, k_x, k_y, k_z)$ subject to a set of constraints equations in the form of finite difference equations. In order to see how the perturbations of $k_x$, $k_y$, and $k_z$ at each grid block affect the generalized travel time shift at each well, a two dimensional vector of Lagrange multipliers is used to adjoin $g$ with the fluid flow equations to from the augmented objective function $J$ as follow:

$$J = \lambda [\lambda_{kx}, \lambda_{ky}, \lambda_{kz}] \quad \text{(B.9)}$$

$$J = g + \sum_{i=0}^{L-1} \lambda_{ki} (F_{m}^{i+1} - A_{m}^{i+1} + A_{m}^{i}) \quad \text{(B.10)}$$

The total differential equation of Eq. (B.10) is performed with respect to the state variables pressure and control variables, $(k_x, k_y, k_z)$

$$dj = dg + \sum_{i=0}^{L-1} \lambda_{ki} [V_{p}(F_{m}^{i+1} - A_{m}^{i})] + B_{T} \xi \quad \text{(B.11)}$$

where $B_{T} \xi$ is calculated from the following equation:

$$B_{T} \xi = [V_{p}(F_{m}^{i+1} - A_{m}^{i})]dp \quad \text{(B.12)}$$

Initially, the pressure values at each grid block is known and the variation of control variables will not have any effect on it, thus,

$$dp^0 = 0 \quad \text{(B.13)}$$

Therefore, any term multiplied by $dp^0$ had to be neglected from Eq. (B.13).

Also, by taking total differentiation of $g$ with respect to state and control variables, the equation will be as follow:

$$dg = \sum_{i=0}^{L-1} \sum_{i=0}^{m-1} [V_{p}g]dF_{m}^{i+1} + [V_{kx}g]dk_{x} + [V_{ky}g]dk_{y} + [V_{kz}g]dk_{z} \quad \text{(B.14)}$$

Substituting Eq. (B.14) into Eq. (B.11):

$$d^2g = \sum_{i=0}^{L-1} \sum_{i=0}^{m-1} \sum_{i=0}^{m-1} [V_{p}g]dF_{m}^{i+1} + [V_{kx}g]dk_{x} + [V_{ky}g]dk_{y} + [V_{kz}g]dk_{z} \quad \text{(B.15)}$$

Eq. (B.15) shows how the changes in $(k_x, k_y, k_z)$ affect the pressure at each grid block at each time step.

The objective now is to find equations that relate the sensitivity of $(g)$ function with respect to control variables (permeability) and to remove the dependency of $J$ function from pressure. Adjoint variables were selected to fulfill this objective. The result is the following adjoint equations:

$$\sum_{i=0}^{L-1} (V_{p}g)^{T} + \sum_{i=0}^{L-1} (V_{p}g)^{T} \quad \text{(B.16)}$$

Similarly, to remove the dependency of pressure change at the end of simulation run, $dp^j$ has to be removed from the system of equations, thus, the following condition is set as follow:

$$\xi^j = 0 \quad \text{(B.17)}$$

Therefore, from Eq. (B.12) and Eq. (B.16). $B_{T} \xi^j = 0 \quad \text{(B.18)}$

From the Eq. (B.17), and Eq. (B.18), the change in the augmented function $j$ in Eq. (B.15) will be:

$$dj = \sum_{i=0}^{L-1} \sum_{i=0}^{m-1} [V_{p}g]dF_{m}^{i+1} + [V_{kx}g]dk_{x} + [V_{ky}g]dk_{y} + [V_{kz}g]dk_{z} \quad \text{(B.19)}$$

In this study, the $(g)$ function is given by:

$$g = \sum_{i=0}^{L-1} (t_{obs} - t_{cal}) \quad \text{(B.20)}$$

where $n_{dp}$ is the number of data point for well $j$, (i) is the data point index at time $t'$. The gradient of the scalar function $g$ in the adjoint system of equations is given by using the gradient of Eq. (B.21) as follow:

$$V_{p}g = \frac{\partial g}{\partial \Delta t} = \frac{\partial \Delta t \xi}{\partial \xi} \quad \text{(B.21)}$$

At grid blocks of producing wells, the partial derivative of the generalized travel time with respect to pressure is given by:

$$\frac{\partial \Delta t}{\partial p} = \frac{1}{n_{dp}} \frac{\partial}{\partial p} \sum_{i=1}^{n_{dp}} (t_{obs} - t_{cal}) \approx - \frac{n_{dp}}{n_{dp}} \frac{\partial \Delta t}{\partial p} \quad \text{(B.22)}$$

**Sensitivity Coefficients Formulation**

In adjoint formulation section, the adjoint variable at each grid blocks has been calculated. This section describes the calculations process for sensitivity coefficient by using the knowns adjoint variables. The sensitivity coefficients calculation equations are given as follow:

$$V_{kx}J^{T} = \sum_{m=0}^{L-1} \sum_{i=0}^{m-1} V_{kx}F_{m}^{i+1}A_{m}^{i+1} = (V_{kx}g)^{T} \quad \text{(B.23)}$$

$$V_{ky}J^{T} = \sum_{m=0}^{L-1} \sum_{i=0}^{m-1} V_{ky}F_{m}^{i+1}A_{m}^{i+1} = (V_{ky}g)^{T} \quad \text{(B.24)}$$
\[
\left( V_i J \right)^T = \sum_{m=0}^{l-1} \sum_{l=0}^{m-1} V_k F_m^{l-1} r_m^{l-1} - \left( V_k \hat{g} \right)^T
\]

(B.25)


This appendix details the FEM formulation of the Poroelastic problem. Assuming off-diagonal components of permeability tensor are zero,

\[
k = \begin{pmatrix} k_{xx} & 0 & 0 \\
0 & k_{yy} & 0 \\
0 & 0 & k_{zz} \end{pmatrix}
\]

\[
\phi c_p \frac{\partial p}{\partial t} - \frac{\partial}{\partial x} \left( \frac{\partial \sigma_{xx}}{\partial x} \right) u + \frac{\partial \left( k_p \frac{\partial p}{\partial x} \right)}{\partial y} + \frac{\partial \left( k_p \frac{\partial p}{\partial y} \right)}{\partial y} + \frac{\partial \left( k_p \frac{\partial p}{\partial z} \right)}{\partial z}
\]

(B.26)

Multiplying both sides of Eq. (B.26) by a trial function, \( w \) and integrating over the domain, \( \Omega \) yields:

\[
\int_{\Omega} \left[ w c_p \frac{\partial p}{\partial t} + \int_{\Omega} \frac{\partial w}{\partial t} \frac{\partial p}{\partial x} \right] \, d\Omega
\]

Using the Green formulae, Eq. (B.2) becomes:

\[
\int_{\Omega} \left[ w c_p \frac{\partial p}{\partial t} + \int_{\Omega} \frac{\partial w}{\partial t} \frac{\partial p}{\partial x} \right] \, d\Omega
\]

where: \( \Gamma \) is the boundary and \( n \) is the outward normal to the boundary. Finite difference method is used to discretise the terms including differentiation with respect to time. Permeability and porosity remain constant with change in time. After rearranging, Eq. (B.28) becomes:

\[
\int_{\Omega} \left[ w c_p \frac{\partial p}{\partial t} + \int_{\Omega} \frac{\partial w}{\partial t} \frac{\partial p}{\partial x} \right] \, d\Omega
\]

In which 'i' and 'i - 1' are current and previous times respectively. Using Galerkin approach (Zienkiewicz and Taylor, 2000k), Eq. (B.29) becomes:

\[
\int_{\Omega} \left[ w c_p \frac{\partial p}{\partial t} + \int_{\Omega} \frac{\partial w}{\partial t} \frac{\partial p}{\partial x} \right] \, d\Omega
\]

\[
\int_{\Omega} \left[ w c_p \frac{\partial p}{\partial t} + \int_{\Omega} \frac{\partial w}{\partial t} \frac{\partial p}{\partial x} \right] \, d\Omega
\]

where:

\[
P^T = (P_1 \ P_2 \ \ldots \ P_n)
\]

(B.31)

\[
\bar{N}_p^T = (N_1 \ N_2 \ \ldots \ N_n)
\]

(B.32)

\[
\bar{N}_e = \begin{bmatrix} N_1 & 0 & N_2 & 0 \\
0 & N_1 & 0 & N_2 \end{bmatrix}
\]

(B.33)

\[
\bar{U}^T = (u_{x1} \ u_{y1} \ u_{x2} \ u_{y2} \ \ldots \ u_{yn})
\]

(B.34)

where 'n' is the total number of nodes. After rearrangement, Eq. (B.30) can be written as:

\[
\bar{U}^T \left[ \bar{P}^i - \bar{p}^{i-1} \right] + \Delta t^e \bar{P}^i - \bar{Q} \left( \bar{U}^i - \bar{U}^{i-1} \right) = \bar{0}
\]

(B.35)

where:

\[
\bar{L} = \sum_{e=1}^{ne} \bar{L}^e
\]

(B.36)

\[
\bar{H} = \sum_{e=1}^{ne} \bar{H}^e
\]

(B.37)

In which:

\[
\bar{Q} = \sum_{e=1}^{ne} \bar{Q}^e
\]

(B.40)

where:

\[
Qe = \alpha \int_{\Omega} \bar{N}_p^e \left( \frac{\partial N^e_u}{\partial x} + \frac{\partial N^e_v}{\partial y} \right) \, d\Omega
\]

(B.41)

Eq. (B.35) in an incremental form can be written as:

\[
\bar{U}_i^e = - \Delta t^e \bar{H}^e \bar{P}^i - \bar{Q} \Delta \bar{U}_i = \bar{F}_i^e
\]

(B.43)

Gaussian quadrature is applied for integration, get:
\[
\int_{\Omega} \left( \frac{W}{S} \tilde{\sigma} \Delta \sigma \right) d\Omega = 0 \tag{B.44}
\]

where:

\[
\bar{W} = \begin{bmatrix}
W(x, y, z) \\
W_{f}(x, y, z) \\
W_{g}(x, y, z)
\end{bmatrix}
\]  \tag{B.45}

And \(w_1\) and \(w_2\) are trial functions. Using Green's identity one obtains:

\[
\int_{\Omega} \left( \frac{W}{S} \tilde{M} \Delta \sigma \right) d\Omega - \int_{\partial \Omega} \frac{\partial}{\partial n} \left( \frac{W}{S} \tilde{M} \Delta \sigma \right) d\Gamma = 0 \tag{B.46}
\]

Where:

\[
\tilde{M} = \begin{bmatrix}
n_{1}n_{0} & n_{0} & n_{0} \\
n_{1} & n_{0} & n_{0} \\
n_{1} & n_{1} & n_{0}
\end{bmatrix}
\]

Rearrange Eq. (B.46), one gets:

\[
\int_{\Omega} \left( \frac{W}{S} \tilde{M} \right)^{T} \frac{\partial U}{\partial \sigma} d\Omega + \alpha \int_{\partial \Omega} \left( \frac{W}{S} \tilde{M} \right)^{T} \frac{\partial \sigma}{\partial \Gamma} d\Gamma = \int_{\Omega} \left( \frac{W}{S} \tilde{M} \Delta \sigma \right) d\Gamma 
\]

Using Galerkin method Eq. (B.48) yields:

\[
\left( \int_{\Omega} \left( \frac{W}{S} \tilde{M} \right)^{T} \frac{\partial U}{\partial \sigma} \right) d\Omega + \alpha \int_{\partial \Omega} \left( \frac{W}{S} \tilde{M} \right)^{T} \frac{\partial \sigma}{\partial \Gamma} d\Gamma = \int_{\Omega} \left( \frac{W}{S} \tilde{M} \Delta \sigma \right) d\Gamma 
\]

Or in a compact form as follow:

\[
\bar{K} \Delta \bar{U} + \bar{Q} \bar{P} = \bar{f}_2 \tag{B.50}
\]

\[
\bar{K} = \sum_{\epsilon=1}^{\infty} \bar{K}^\epsilon \tag{B.51}
\]

\[
\bar{K}^\epsilon = \int_{\Omega} \left( \frac{W}{S} \tilde{M}^\epsilon \right)^{T} \frac{\partial U}{\partial \sigma} \frac{\partial \sigma}{\partial \epsilon} d\Omega \tag{B.52}
\]

\[
\bar{f}_2 = \sum_{\epsilon=1}^{\infty} \bar{f}_2^\epsilon \tag{B.53}
\]

\[
\bar{f}_2 = \int_{\Omega} \left( \frac{W}{S} \tilde{M} \Delta \sigma \right) d\Omega \tag{B.54}
\]

Eq. (B.42) and Eq. (B.50) are the final finite element equations to be simultaneously solved as a system of linear equations which is as follows:

\[
\begin{bmatrix}
\bar{L} + \Delta \bar{H} & -\bar{Q} \\
\bar{Q}^{T} & \bar{K}
\end{bmatrix}
\begin{bmatrix}
\bar{P} \\
\bar{J}_2
\end{bmatrix}
= \begin{bmatrix}
\bar{f}_1 \\
\bar{f}_2
\end{bmatrix} \tag{B.55}
\]
permeability changes due to cold fluid circulation in fractured geothermal re-
servoirs. Groundwater.

Gilman, J., 1986. An efficient finite-difference method for simulating phase segre-
gation in the matrix blocks in double-porosity reservoirs. SPE Reserv. Eng. 1
(04), 403–413.

Learn. 3 (2), 95–99.

Gong, B., Karimi-Fard, M., Durlufsky, L.J., 2008. Upscaling discrete fracture char-
acterizations to dual-porosity, dual-permeability models for efficient simul-
ation of flow with strong gravitational effects. SPE J. 13 (1), 58.

stone Formation Using Stochastic Inverse Approaches. Lawrence Berkeley Lab.,
CA, United States.


Gutmanis, J.C., i Oró, L.A., 2015. Application of pyrenean fractured carbonate out-
crops for subsurface reservoir characterization. In: Proceedings of the 77th

models. SPE J. 10 (3), 312–323, SPE-81503-PA.

stochastic models of field-scale fractures: methodology and case study. In: Pro-
cceedings of SPE Annual Technical Conference and Exhibition. Society of Petrol-
eum Engineers, January.

method to use transient testing results in reservoir simulation. In : Pro-
cceedings of SPE Annual Technical Conference and Exhibition. Society of Petroleum
Engineers, January.

Kazemi, H., 1969. Pressure transient analysis of naturally fractured reservoirs with

Kuchuk, F.J., Biryukov, D., 2012. Transient pressure test interpretation from con-
tinuously and discretely fractured reservoirs. In: Proceedings of SPE Annual
Technical Conference And Exhibition. Society of Petroleum Engineers, San
Antonio, TX.

Landa, J., Horne, R., Kamal, M., Jenkins, C., 2000. Reservoir characterization con-
strained to well-test data: a field example. SPE Reserv. Eval. Eng. 3 (04),
325–334.

Landereau, P., Noetinger, B., Quintard, M., 2001. Quasi-steady two-equation models
for diffusive transport in fractured porous media: large-scale properties for

Liu, Ning, Oliver, D.S., 2004. Automatic history matching of geologic facies. SPE J.
19 (04), 429–436.

permeability of grid blocks used in the simulation of naturally fractured re-
Conference.

Lufth, S.M., 2005. Fractured reservoir analysis using modern geophysical well tech-
nologies: application to basement reservoirs in Vietnam. In: Harvey, P.K.,
Brewer, T.S., Pezard, P.A., Petrov, V.A. (Eds.), Petrophysical Properties of Crys-

Mavor, M.J., Ley, H.C., 1979. Transient Pressure Behaviour of Naturally Fractured
Reservoirs SPE-79777-MS. http://dx.doi.org/10.2118/79777-MS.

Morton, K.L., 2012. Integrated interpretation for pressure transient tests in dis-
the EAGE Annual Conference & Exhibition Incorporating SPE Europe. 4–7 June.
Copenhagen, Denmark.

80 (10), 1565–1585.

sional Publishing, USA.

using continuous-time random walks methods. Transp. Porous Media 39,
315–337.

Noetinger, B., 2014. A quasi steady state method for solving transient Darcy flow in
complex 3D fractured networks accounting for matrix to fracture flow. J.

Noetinger, B., 2015. A quasi steady state method for solving transient Darcy flow in
complex 3D fractured networks accounting for matrix to fracture flow. J.

35 (4), 483–495.

Odeh, A.S., Babu, D., 1990. Transient flow behavior of horizontal wells pressure
drawdown and buildup analysis. SPE Form. Eval. 5 (01), 7–15.

Comput. Geosci. 15 (1), 185–221.

Oliver, D.S., 1994. Multiple Realizations of the Permeability Field from Well Test
Data. Society of Petroleum Engineers, Richardson, TX, United States.

Ortega, O.J., Marrrett, R.A., Laubach, S.E., 2006. A scale-independent approach to
fracture intensity and average spacing measurement. AAPG Bull. 90 (2),
193–208.

seismic on integrated naturally fractured reservoir characterization. In: Pro-
ceedings of SPE Asia Pacific Conference on Integrated Modelling for Asset

with an effective permeability tensor and its application to naturally fractured
reservoirs. Energy Sources 24 (6), 531–542.

Pride, S.R., Berryma, J.C., 2003. Linear dynamics of double-porosity dual-perme-
68 (3), 036603.


Water Resour. 29 (7), 1020–1036.

sachusetts Institute of Technology. Earth Resources Laboratory, USA.


5 (6), 1273–1288.

Massachusetts Institute of Technology. Earth Resources Laboratory.

Suzuki, S., Dally, C., Caers, J., Mueller, D., 2005. History Matching of Naturally Frac-
tured Reservoirs Using Elastic Stress Simulation and Probability Perturbation


Technol. 23 (5–6), 693–709.

Vasco, D.W., Seongsik, Y., Datta-Gupta, A., 1999. Integrating dynamic data into high-
resolution reservoir models using streamline-based analytic sensitivity coeffi-
cients. SPE J. 4 (04), 385–399.

CO2 storage monitoring or oil recovery history matching. Oil Gas Sci. Technol.

Will, R., Archer, R.A., Dershowitz, W.S., 2005. Integration of seismic anisotropy and
reservoir properties data for characterization of naturally fractured re-
servoirs using discrete fracture network models. SPE Reserv. Eval. Eng. 8 (02),
132–142.

Wu, Z., Datta-Gupta, A., 2002. Rapid history matching using a generalized travel-
time inversion method. SPE J. 7 (02), 113–122.

Methods. Society of Petroleum Engineers, Richardson, TX, United States.
