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Pre- and Post-Coding for Capacity Enhancement in MIMO Systems

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Abstract

A linear scheme at the transmitter (pre-coder) and the receiver (post-coder) is proposed to improve the performance of MIMO systems under a specific configuration for transmitting and receiving antennas over fading channels. This is achieved by generating a linear pre- and post-coder matrix that depends on the correlation matrix of the channel state information (CSI). Through MATLAB simulation, the results show that the use of appropriate coder matrix enhances the MIMO channel capacity and reduces the sensitivity on CSI.

Keywords: MIMO; pre-coder; post-coder; channel capacity

1. Introduction

Demands for capacity in wireless communications, driven by cellular mobile, internet and multimedia services, have been rapidly increased worldwide. On the other hand, the available radio spectrum is limited and the communication capacity needs cannot be met without a significant increase in communication spectral efficiency. Advances in coding made it feasible to approach the Shannon capacity limit in systems with a single antenna link.

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Further significant advances in spectral efficiency are available through increasing the number of antennas at both the transmitter and the receiver sides which is known as a MIMO system[1, 2].

A MIMO channel can be realized with multi-element array antennas, of particular interest are propagation scenarios in which individual channels between certain pairs of transmit and receive antennas are modeled by an independent flat fading process, which is realistic for environments with a large number of scatters, where a signal transmitted from every individual antenna appears uncorrelated at each of the receive antennas[1].

Based on the statistical information of the channel, the use of diagonal pre-coding can greatly improve the system performance[3, 4]. For channel diagonalization, it has been shown[2] that the MIMO channel can be decomposed into parallel eigen sub-channels, or equivalently eigen-modes, by means of singular value decomposition (SVD). Moreover, a general framework for pre- and post-coder designs for MIMO systems using partial channel knowledge on transmit and receive correlation matrices at the transmitterhave been presented by Bahrami and Le-Ngoc[5]. It has been shown that the optimal linear pre-coder for any uncoded and coded MIMO system based on the minimum mean square error (MMSE) or ergodic capacity criterion is an eigen beam-former that transmits the signal along eigenvectors of the transmit correlation matrix and the optimal post-coder is the inverse of the eigenvector. Based on the eigen-values of both transmit and receive correlation matrices, power loading across the eigen-beams is determined by water filling algorithm; which is a power allocation scheme used to rearrange the power at each sub channel of the MIMO system at different signal to noise ratio (SNR)[6].

In addition, transceivers design based on geometric mean decomposition (GMD) for identical parallel channels has been obtained and then improved[7, 8], while new channel decomposition strategy (called LDH^H) has been presented and used for low complexity MIMO pre-coder design[9]. On the other hand, employing a pre-coder matrix with full rank, have been extensively investigated for the subspace-based channel estimation[10]. Jung, Hwang and Choi have proposed a multi-mode pre-coding scheme based on the interference minimization[11] in order to maximize the total system capacity based on mode selection.

In this paper, we present a search algorithm to design linear MIMO pre- and post-coders with a specific configuration of transmitting and receiving antennas over flat fading sub-channels. Simulation results show that our proposed pre- and post-coder enhances the system capacity under a set of power constrain with partial CSI knowledge at different patterns of correlation in the channel matrix.

2. System Description

Consider the MIMO communication model depicted in Fig. 1. The input bit-stream is pre-coded and modulated to generate at least one output symbol stream then it is launched into the MIMO channel using N_T separate antennas. Afterwards, the post-coder receives this stream and performs a decoding process to regenerate the transmitted data stream.
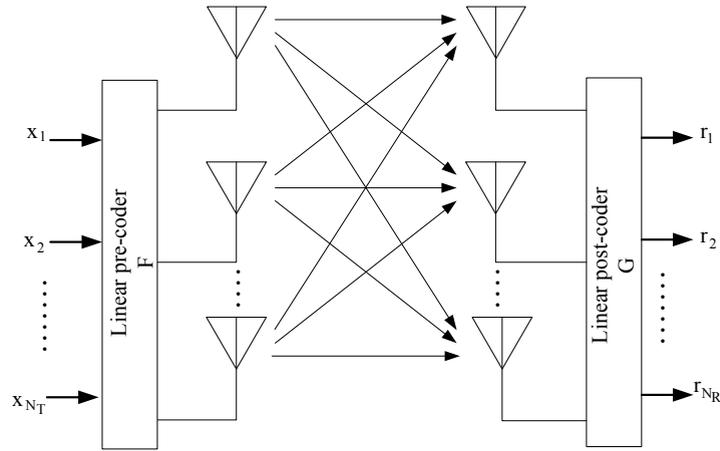
Fig. 1. MIMO Model.

Consider an ergodic flat fading Rayleigh channel with N_T transmit and N_R receive antennas, which can be described in a vector form as follows[12]:

$$\mathbf{r} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

where $\mathbf{r}$, $\mathbf{H}$, $\mathbf{x}$, and $\mathbf{n}$ are the received signal vector, the MIMO channel matrix, the transmit symbol vector, and the noise vector, respectively. To maximize the mutual information of the system, it is assumed that $\mathbf{x}$ is circularly symmetric Gaussian[1, 12], while the short-term power is constrained by P to obtain:

$$P = \text{tr}(\mathbf{R_{xx}}) \tag{2}$$

The channel matrix $\mathbf{H}$ is assumed to have independent and identically distributed (i.i.d.) zero-mean complex Gaussian entries (so that the channel amplitude follows Rayleigh fading). Likewise, $\mathbf{n}$ have zero-mean complex Gaussian i.i.d. entries, each of them with variance of σ^2.

In many applications, transmit and/or receive antennas can be correlated. In such a case, antenna correlations tell us about the available spatial diversity in a MIMO channel. If the antennas are highly correlated, very little spatial diversity gain can be extracted from the channel, and vice versa.

For an $N_R \times N_T$ MIMO channel, the $N_T \times N_T$ transmit antenna correlation matrix is given by[13]:

$$\mathbf{R}_{a,T} = \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,N_T} \\ \rho_{2,1_T} & 1 & \cdots & \vdots \\ \cdots & \cdots & \cdots & \vdots \\ \rho_{N_T,1} & \cdots & \cdots & 1 \end{pmatrix} \tag{3}$$

where $\rho_{i\,j}$ is the correlation coefficient between the ith and j[th] transmit antennas. Similarly, the $N_R \times N_R$ receive antenna correlation matrix $\mathbf{R}_{a,R}$ can be defined.

The correlation matrix of the MIMO channel in the case of correlated antennas is the Kronecker product of the spatial correlation matrices at the transmitter and receiver sides[5], which is given by:

$$\mathbf{R}_{mimo} = \mathbf{R}_{a,T} \otimes \mathbf{R}_{a,R} \tag{4}$$

The channel model; including antenna correlations; could be written as[14], where $\mathbf{H}_c$ is the correlated MIMO channel and $\mathbf{H}$ is the uncorrelated MIMO channel:

$$\mathbf{H}_c = \mathbf{R}_{a,T}{}^{1/2} \, \mathbf{H} \, \mathbf{R}_{a,R}{}^{1/2} \tag{5}$$

The effect of channel correlation on the capacity depends on the known information about the channel at the transmitter and receiver sides. In addition, if the antenna correlations are high, the correlation matrix and hence $\mathbf{H}_c$ will have a lower rank leading to a limited capacity gain.

For a flat fading channel with pre- and post-coders, the system is characterized by:

$$\mathbf{r} = \mathbf{GHFx} + \mathbf{Gn} \tag{6}$$

The capacity of the MIMO system using a linear pre and post-coder is defined by[14-17]:

$$C = E\left\{ \log_2 \det\left(\mathbf{I} + \frac{P}{N_T \sigma^2} \mathbf{Q} \right) \right\} \tag{7}$$

Where $\mathbf{Q}$ is the Wishart matrix given as:

$$\mathbf{Q} = \begin{cases} \mathbf{H_{eq} H_{eq}^H} & N_R < N_T \\ \mathbf{H_{eq}^H H_{eq}} & N_R \geq N_T \end{cases} \tag{8}$$

And

$$\mathbf{H_{eq}} = \mathbf{GHF} \tag{9}$$

An enhancement in the MIMO capacity could be achieved by increasing the determinant of the equivalent channel[18] and diagonalization of the channel matrix which means the transformation of the MIMO channel into separate and independent single input single output (SISO) channels, so there is no co-channel interference (CCI), which is the main goal of this paper.

3. Performance Evaluation

As explained before, our pre- and post- coder design considers the risk of correlated channel matrix under line of sight propagation, which is not fully correlated channel. Therefore, we study several scenarios of correlated MIMO channel matrix, in which the correlation may exist in one or more rows or/and columns of $\mathbf{H}$. The amount of correlation is also varying to make our pre- and post-coder operates at different amounts of correlations.

Our design assumes partial channel knowledge at the transmitter side, which needs a slow feedback link, i.e., the change of the correlation matrix is much slower than that of the channel matrix. As the goal of this research to find a matrix that can be used with correlated channels, we start with a condensed searching algorithm that aims at finding the best matrices to be used for different types of correlated channels with various amounts of correlations i.e. between 95% and 75% correlation. Eventually, we consider that errors in estimating the correlation matrix may

occur. Therefore, we need to search for a coder that is stable and can properly work even if the real correlation is higher or less than the values used in the design.

Simulation for the proposed scheme has been carried out using MATLAB. In our experiments, an input SNR vector is defined and a variable vector is also stated in order to change the correlation coefficient and the pattern of correlation in the random channel that is also generated under line of sight propagation. At each iteration, channel capacity and condition number are calculated then the mean values of those are taken at the end of the simulation run. Afterwards, a graph is plotted to show the old and new capacity for correlated MIMO channel compared with the capacity of the uncorrelated MIMO channel.

To generalize our design to be valid for any number of transmitting and receiving antennas, we should generate pre- and post-coder matrices that increase the determinant of the equivalent MIMO system by solving the rank-deficient or near singular matrix. According to MIMO channel capacity that is represented in equation 7, it is clear that we should use a coder matrix that has the following specifications:

- The rank of $\mathbf{F}$ and $\mathbf{G}$ equals the minimum number of transmit and receive antennas under consideration; that is the coder is a full rank matrix.

- $\mathbf{F}$ and $\mathbf{G}$ should be non-singular matrix, so that after multiplying the channel with the coder matrix, we obtain a non-singular $\mathbf{H_{eq}}$ and when calculating the capacity we get an enhancement due to the increase in the determinant.

- $\mathbf{F}, \mathbf{G}$ and $\mathbf{H_{eq}}$ should have a well condition number[19]. The condition number is used to denote the spread of eigenvalues and it is defined as $(\lambda_{min}/\lambda_{max})$. A unity condition number means the channel matrix is orthogonal. A small condition number means ill conditioned channel.

In our proposed scheme, we consider a 3×3 MIMO system with a complex Gaussian channel matrix to insure direct line of sight propagation. As a result, the mean of the channel equals one and its variance equals one; half at the real part and half at the imaginary part of the complex MIMO channel. The signal components arriving at the receiver may experience correlation due to limited distance between antenna elements or some scattering conditions, thus we deal with variable correlated MIMO channels at various scenarios of correlation to find the best $\mathbf{F}$ and $\mathbf{G}$ matrices in order to enhance the capacity of the correlated MIMO channel with a total power constraint; that is the addition of the coder does not enhance the system power $\left(\text{tr}\left(\mathbf{H}_{eq}^{H}\mathbf{H_{eq}}\right) \leq P\right)^{20}$.

We first study the capacity versus several values of SNR for highly row correlated channel. We examine four different channel states: uncorrelated channel, correlated channel, correlated channel with pre-coder matrix, and fully correlated channel. As Fig. 2 shows, fully correlated channel with pre-coder matrix outperform other channels since the three columns entries are exactly the same and the MIMO capacity will be equal the SISO system capacity. On the other hand, with post-coder matrix, an enhancement in the MIMO channel capacity is achieved as shown in Fig.3.

Fig. 4 shows the enhancement in the capacity for highly correlated row channel matrix with certain encoder matrix as shown below where it is used as pre-coder and post-coder matrices.

$$\mathbf{G} = \mathbf{F} = \begin{bmatrix} 0.3j & -0.4j & 0.8j \\ 0.9j & 0.5j & -0.9j \\ -1.5j & 1.5j & 0.1j \end{bmatrix} \tag{13}$$
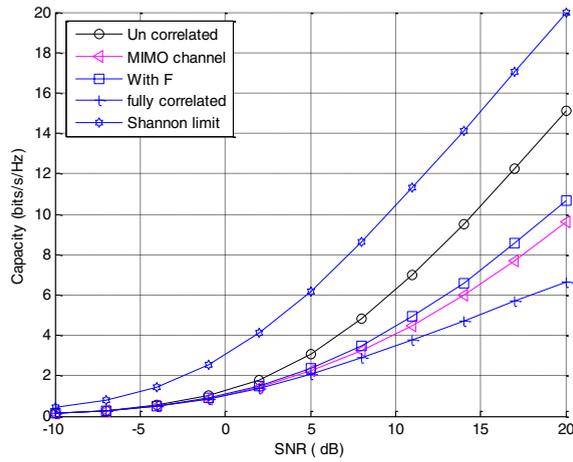
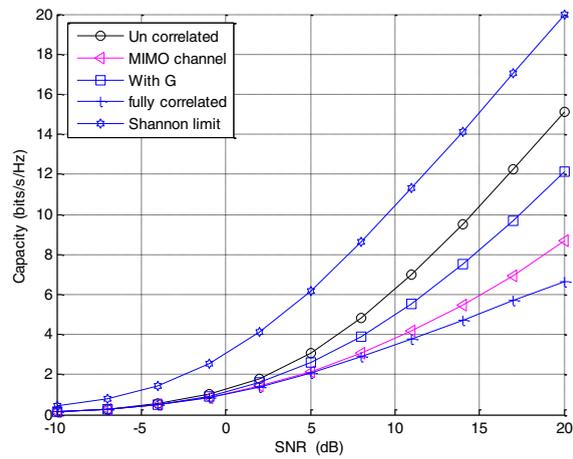Fig. 2. fully row correlated channel with pre-coder

Fig. 3. fully row correlated channel with post-coder

In Fig. 5, we examine the use of various $\mathbf{F}$ and $\mathbf{G}$ when a high correlation between the first and third row of the channel matrix as follows:

$$\mathbf{F} = \begin{bmatrix} 0.3j & -0.4j & 0.8j \\ 0.5j & 0.5j & -0.5j \\ -1.5j & 1.5j & 0.1j \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0.3j & -0.4j & 0.8j \\ -1.5j & 1.5j & 0.1j \\ 0.5j & 0.5j & -0.5j \end{bmatrix} \tag{14}$$
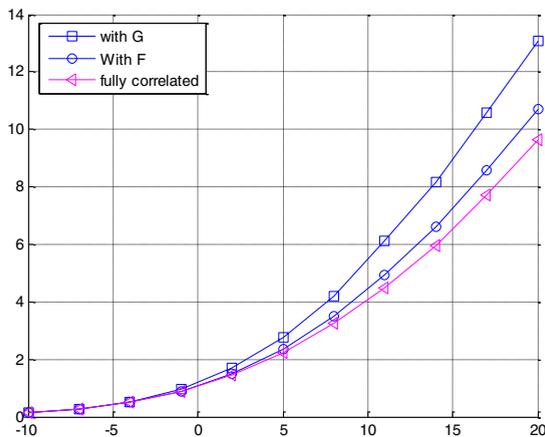
Fig. 4. pre- and post- coder capacity for highly correlated rows of a MIMO channel
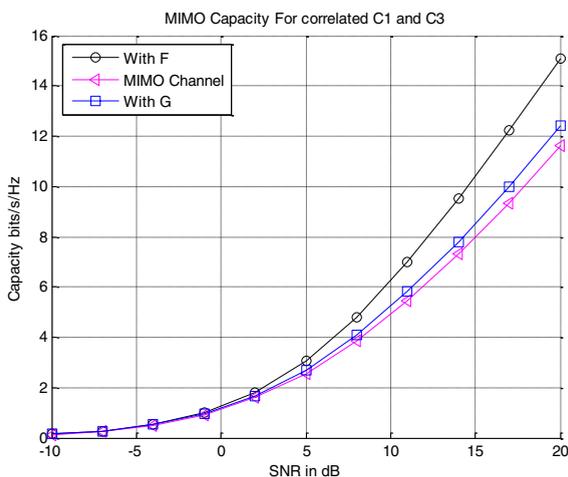

Fig. 5. pre- and post- coder capacity for 95% correlation between the first and third rows

4. Conclusion

In this paper we proposed a design for a pre- and post-coder of a 3×3 MIMO system. The proposed scheme is designed with the aim of maximizing the capacity for MIMO systems through decorrelating the channel matrix, or, in other words, to reduce the effect of high correlations in the channel matrix which degrades the MIMO system capacity so that the assumed capacity gains cannot be achieved.

The proposed design will search to find the optimal linear coder based on row or column correlation of the channel matrix so a fast feedback channel from the receiver to the transmitter is not required. The proposed design has been conducted using certain searching algorithm. Simulation results have been showed that the proposed pre- and post-coders have been achieved variable capacity enhancements for various correlation types. The design has been fulfilled the stability requirements even if the error in estimating the correlation matrix occurs.

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