The effect of beam depth on the shear behavior of reinforced concrete beams externally strengthened with carbon fiber–reinforced polymer composites

Rajai Z Al-Rousan¹,² and Mohsen A Issa³

Abstract
The primary objective of this article is to study the effect of beam depth on the performance of shear-deficient beams externally strengthened with carbon fiber–reinforced polymer composites. The investigated parameters include overall behavior up to failure, the onset of the cracking, crack development, and ductility. The experimental results showed that externally bonded carbon fiber–reinforced polymer increased the shear capacity of the strengthened reinforced concrete beams significantly depending on the variables investigated. The use of carbon fiber–reinforced polymer composites is an effective technique to enhance the shear capacity of reinforced concrete beams. For the beams tested, as the depth of the reinforced concrete beams was increased from 225 to 450 mm which is equivalent to \( a/d \) ratio of 2.7–1.2, respectively, there was corresponding 15%–19% increase in contribution of the carbon fiber–reinforced polymer strips in terms of ultimate load. The impact of the beam depth is more pronounced on the ultimate load than the corresponding deflection of the control and strengthened beams. The results indicated that the beam depth has an influence on the angle at which primary cracking angle varied from 33, 44, 50, and 54 for beam of \( a/d \) of 2.7, 1.9, 1.5, and 1.2, respectively. If the shear crack crossed carbon fiber–reinforced polymer strips above its development length, the carbon fiber–reinforced polymer strips could not be expected to reach its ultimate strength and was thus only partially effective as observed in beams with different depths.

Keywords
beam depth, carbon fiber–reinforced polymer composites, externally, reinforced concrete beams, shear behavior, strengthened

Introduction
The deterioration of civil engineering infrastructures such as buildings, bridge decks, girders, offshore structures, and parking structures is mainly due to aging, poor maintenance, corrosion of steel reinforcement, defects in construction/design, demand in the increased service loads, exposure to harmful environments and damage in case of seismic events, and improvement in the design guidelines. These deteriorated structures are deficient to take the load for which they are designed. A large number of structures constructed in the past using the older design codes in different parts of the globe are structurally unsafe according to the new design codes and hence need upgradation of the existing structures. Also, increases in traffic volume, traffic loads, and corrosion-induced deterioration are necessitating significant expenditures to strengthen and rehabilitate existing structures (Noel and Soudki, 2011). Of the 163,000 single-span concrete bridges in the United States, 23% are considered structurally deficient or functionally obsolete (Mabsout et al., 2004).

The shear failure of a reinforced concrete (RC) beam is distinctly different from the flexural one in that the flexural is ductile in nature, whereas the shear one is brittle and catastrophic. When the RC beam is deficient in shear, or when its shear capacity is less than the flexural capacity after flexural strengthening, shear strengthening of the beam must be considered. It is critically important to examine the shear capacity of RC beams which are intended to be strengthened in flexure. Fiber-reinforced polymers (FRPs) have emerged as promising material for rehabilitation of...
existing RC structures and strengthening of the new civil engineering structures because of their several advantages such as high strength-to-weight ratio, high fatigue resistance, flexible nature, ease of handling, and excellent durability. Therefore, the recent advancement of FRP composites as a repair and strengthening material for RC beams, slabs and columns in structural engineering applications has increased over the past 20 years (American Concrete Institute (ACI), 2007; Bank, 2006; Cao et al., 2005; Chaallal et al., 1998; Khalifa et al., 1998; Triantafillou, 1998; Triantafillou and Antonopoulos, 2000). The non-corrosive properties and high strength-to-weight ratio of FRP materials make them a very viable repair material and can result in longer service life of structures (Noel and Soudki, 2011). The advancement of new materials and technologies has led researchers to investigate various designs and materials to increase the strength of shear critical RC beams (Ascione and Feo, 2000; Garden and Hollaway, 1998; Sim et al., 2005; Taljsten and Elfgren, 2000; Yao et al., 2005). Various materials, configurations, wrapping techniques, and mechanical anchors have been explored to increase the capacity of existing RC members and postpone or delay the debonding process in externally bonded FRP members (Adhikary and Mutsuyoshi, 2004; Bousselham and Chaallal, 2008; Bukhari et al., 2010; Chaallal et al., 2002; Chen et al., 2010, 2012; Chen and Teng, 2003; Deniaud and Cheng, 2001; Khalifa and Nanni, 2000; Kim and Smith, 2009a, 2009b; Orton et al., 2008; Quinn, 2009; Smith and Kim, 2008). The beam depth, shear span-depth ratio, and beam size parameters had a strong impact on the behavior of the RC beams strengthened in shear with FRP Strips (Islam et al., 2005; Leung et al., 2007; Li and Leung, 2015).

It is observed from the existing literature that the potential use of carbon fiber–reinforced polymer (CFRP) strips in strengthening the RC rectangular beams is reported without taking into account the effect of beam depth. Based on the critical review of the existing literature, the main objective of this work is to study the effect of beam depth on the shear behavior of RC beams externally strengthened with CFRP composites.

Description of experimental program

Ingredient properties

Concrete. All the specimens were made from the same batch of normal weight concrete and conventional fabrication and curing techniques were used. The maximum size of coarse aggregate was 19 mm crushed limestone. Type I Portland cement and admixture were used for all concrete mixes. Table 1 shows the mixture design proportions of concrete used in this study. The concrete mix had a slump in the range of 75–125 mm. Twelve 150 mm × 300 mm concrete cylinders were cast along with each group and cured in the moisture room. The compressive strength of concrete $f'_{c}$ was determined by testing standard concrete cylinders that were taken from the same mix batch at the time of testing the specimens.

Carbon fiber sheets. One type of carbon fiber sheet was used in the research program depending on the manufacturers. This type was the carbon fiber unidirectional sheet in the form of tow sheet. The carbon fiber products come in 500-mm-wide rolls of continuous fiber that can be cut into appropriate lengths. The provided and tested property of the carbon fiber tow sheet is shown in Table 2.

Reinforced concrete beam details

Eight rectangular RC beams were casted and tested with cross sections of 150 mm × 225 mm, 150 mm × 300 mm, 150 mm × 375 mm, and 150 mm × 450 mm and a total length of 1500 mm. The beams were reinforced with 2ф8 bars at the top and 3ф15 bars at the bottom and without stirrups for all the specimens. The design choices were made to ensure that shear failure would occur in the beams. Four beams were tested as control beam without strengthening and

Table 1. Mixture design proportions of concrete.

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Mix proportions (for 1 m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>357 kg</td>
</tr>
<tr>
<td>Coarse aggregate</td>
<td>1026 kg</td>
</tr>
<tr>
<td>Fine aggregate</td>
<td>645 kg</td>
</tr>
<tr>
<td>Silica fume</td>
<td>18 kg</td>
</tr>
<tr>
<td>Fly ash</td>
<td>71 kg</td>
</tr>
<tr>
<td>Water</td>
<td>161 kg</td>
</tr>
<tr>
<td>RB 1000 super plasticizer</td>
<td>90 fl oz</td>
</tr>
<tr>
<td>MB-VR air-entraining</td>
<td>15 fl oz</td>
</tr>
</tbody>
</table>

Table 2. Mechanical properties of CFRP.

<table>
<thead>
<tr>
<th>Property</th>
<th>Provided amount</th>
<th>Tested amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate strength</td>
<td>4275 MPa</td>
<td>–</td>
</tr>
<tr>
<td>Design strength</td>
<td>3790 MPa</td>
<td>3920 MPa</td>
</tr>
<tr>
<td>Yielding modulus</td>
<td>228 GPa</td>
<td>231 GPa</td>
</tr>
<tr>
<td>Ultimate strain</td>
<td>0.0168 mm/mm</td>
<td>0.017 mm/mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.165 mm</td>
<td>0.165 mm</td>
</tr>
</tbody>
</table>
four beams were strengthened with CFRP composites. Figure 1 shows the reinforcement and the CFRP strip configurations for all the beams.

The CFRP sheets were applied to four beams after 28 days of concrete casting. The CFRP sheets/strips of the required length were cut and bonded to the web and/or tensile face of the beams. The details and number of layers of carbon fiber for all beam specimens are shown in Table 3. In the beam designation of Table 3, the first letter “B” indicates the beam specimen; 2.7,
1.9, 1.5, and 1.2 stand for the shear span over effective depth \((a/d)\). The letters: N stands for no strengthened; U90 for 90° U-wrap, and ST for strip wrap followed by the number of layers of CFRP strips.

**Test set-up and instrumentation**

All specimens were tested as simply supported in a specially designed built-up rigid steel frame. A hydraulic jack was used to apply a concentrated load through a hydraulic cylinder on a spread steel beam to produce two-point loading condition to generate a constant moment region at mid-span. Three types of instruments were used in the tests: linear variable differential transformer (LVDT), strain gages, and load cell. Four LVDTs were used: one to monitor the vertical displacement, the LVDT was located at mid-span; two to monitor the crack opening, the LVDTs were located at critical shear stress on both sides; and finally, one to measure the concrete strain at top face. For each specimen, at least three strain gages were attached directly to the FRP strips to monitor the strain during loading. A load cell was used to measure the applied load throughout the tests and a TDS 302 data acquisition system to collect the data of the strain gages, LVDTs, and the load. During loading, the formation of cracks on the sides of the beams was also marked and recorded. The RC shear beam test set-up is shown in Figure 2.

**Experimental results and discussion**

**Mode of failure**

All control specimens without shear reinforcement exhibited an initial flexural crack at the center of the specimen and subsequent flexural cracks away from that section. As the applied load was increased, one of the flexural cracks extended into a diagonal crack near one of the supports, or a diagonal crack formed abruptly at the mid height of the beam within the shear span area. After the formation of the diagonal crack, failure occurred by splitting along the tension reinforcement. The ultimate failure mode was shear failure and the angle of the diagonal shear crack is varied from 33, 44, 50, and 54 for beam of \(a/d\) of 2.7, 1.9, 1.5, and 1.2, respectively. A representative cracking pattern is shown in Figure 3.

Figure 4(a) shows the representative cracking patterns of strengthened tested beam (B2.7U90ST1). The initial flexural crack started at the center of the beam within the constant moment region at 34.2 kN. Beyond this load, cracks extended toward the top fiber, and additional flexural cracks were developed throughout the beam length. At 112.5 kN, a 33-degree angle shear crack developed independently of the existing flexural cracks in the center of the shear span. With further load increase, the Strip No. 4 was debonded at 139.2 kN followed by the debonding of Strip No. 3 at 142.8 kN. The beam failed successively at 149.0 kN after the debonding of Strip No. 5 as shown in Figure 4(a) and Table 3. Debonding of the CFRP strip is a delamination between the strip-adhesive-concrete at the strip-end region of the strengthened beam. This failure was a result of the maximum stresses in the adhesive being not greater than the bonding strength between strip-adhesive-concrete at the strip-end region.

Figure 4(b) shows the representative cracking patterns of strengthened tested beam (B1.9U90ST1). The initial flexural cracks started at 42.3 kN at the center of the beam within the constant moment region. Beyond this load, cracks extended toward the top fiber. Additional flexural cracks were developed throughout the beam length. At 124.5 kN, a 44-degree angle shear crack developed independently of the existing flexural cracks in the center of the shear span. With further increase in load, Strip No. 4 started to debond at 250.0 kN followed by the debonding of Strip No. 5 at 251.0 kN. The beam failed successively in shear at 252.2 kN after the debonding of Strip No. 2 which is closest to support point as shown in Figure 4(b) and Table 3.
Figure 4(c) shows the representative cracking patterns of strengthened tested beam (B1.5U90ST1). The initial flexural cracks started at 51.6 kN at the center of the beam within the constant moment region. Beyond this load, cracks extended toward the top fiber. Additional flexural cracks were developed throughout the beam length. At 133.4 kN, a 50-degree angle shear crack developed independently of the existing flexural cracks in the center of the shear span. With further increase in load, Strip No. 4 started to debond at 293.6 kN followed by the debonding of Strip No. 5 at 300.2 kN. The beam failed successively in shear at 300.2 kN as shown in Figure 4(c) and Table 3.

Figure 4(d) shows the representative cracking patterns of strengthened tested beam (B1.2U90ST1). The initial flexural cracks started at 84.5 kN at the center of the beam within the constant moment region. Beyond this load, cracks extended toward the top fiber. Additional flexural cracking developed throughout the beam length. At 164.6 kN, a 54-degree angle shear crack developed independently of the existing flexural cracks in the center of the shear span. With further increase in load, Strip No. 4 started to debond at 476.0 kN followed by the debonding of Strip No. 5 at 484.8 kN. The beam failed successively in shear at 484.4 kN as shown in Figure 4(d) and Table 3.

Ultimate load capacity
Figure 5(a) shows the normalized ultimate load and ultimate deflection of the strengthened beams with respect to the control ones with a depth of 225 mm.
Inspection of Figure 5(a) reveals that the ultimate load capacity of the strengthened beams increased by 69%, 102%, and 225% with the increase in the beam depth by 33% (300 mm), 67% (375 mm), and 100% (450 mm), respectively. The impact of the beam depth on the ultimate load of the strengthened beams is equal to the control ones. Also, Figure 5(a) shows that the ultimate deflection of the strengthened beams increased by 42%, 64%, and 122% with the increase in the beam depth by 33% (300 mm), 67% (375 mm), and 100% (450 mm), respectively. While, the impact of the beam depth on the ultimate deflection of the strengthened beams is equal to one-third the control ones with a percentage of 14%, 30%, and 41% for the beam depth of 300, 375, and 450 mm, respectively. In addition, inspection of Figure 5(b) reveals that the normalized ultimate load capacity of the strengthened beams with respect to the control beams is 15%, 16%, 18%, and 18% for beams with $a/d$ ratio of 2.7, 1.9, 1.5, and 1.2, respectively. Whereas, the normalized ultimate deflection capacity of the strengthened beams with respect to the control beams is 80%, 44%, 25%, and 21% for beams with $a/d$ ratio of 2.7, 1.9, 1.5, and 1.2, respectively.

### Load–deflection behavior

Figure 6 shows the load–deflection curves for tested beams. All strengthened beams exhibited almost linear load–deflection relationships up to the load of 129.9
201.1, 222.8, and 389.2 kN that equals to the failure load of B2.7N0, B1.9N0, B1.5N0, and B1.2N0, respectively, beams. This indicates that the CFRP started to carry the load after the formulation of the diagonal shear crack. Inspection of Figure 6 shows that the ultimate load capacity of the beams increased with the increase in effective depth as well as the increase in stiffness can be observed from the rotation angle of the elastic stage curve of the tested beams. In addition, Figure 6 shows that the ductility of the beam increased with the increase in beam depth which is the exact mirror of the mode of failure.

**Concrete compressive strain**

Figure 7 shows the relationship between the load and concrete compressive strain for tested beams. Inspection of Figure 7 revealed that the compressive strain in the concrete increased with the increase in the effective depth (d). Also, Figure 7 shows that the concrete compressive strain of the beams increased with the increase in effective depth. The strengthened RC beam (B1.2U90ST1) CFRP sheet registered the highest strain.

**CFRP tensile strain**

Figure 8 shows the relationship between the load and CFRP sheet tensile strain for tested beams. According to Figure 8, the tension strain in the sheet was initiated after the diagonal shear crack starts to formulate at loads of 98.7, 157.9, 180.6, and 192.2 kN for B2.7U90ST1, B1.9U90ST1, B1.5U90ST1, and B1.2U90ST1 beams, respectively. Figure 8 also shows...
that the development of strains becomes sluggish around a load of 77.8 kN for B2.7U90ST1, B1.9U90ST1, and B1.5U90ST1 beams. Inspection of Figure 8 reveals that the sheet tensile strain increased with the increase in effective depth ($d$). It is also observed that the development of strain slows down as the effective depth decreases. At ultimate load, the ultimate tensile strains on sheets were 3165, 7720, 8615, and 9640 $\mu$e for B2.7N0, B1.9N0, B1.5N0, B1.2N0, B2.7U90ST1, B1.9U90ST1, B1.5U90ST1, and B1.2U90ST1, respectively, after the formulation of diagonal shear crack. Figure 9 also shows that the development of crack width also becomes sluggish around 0.25 mm in all beams. It can be observed that the crack developed at a slower rate with the increase in effective depth ($d$). At ultimate load, the ultimate crack width is 1.78, 2.56, 2.81, 3.07, 1.14, 1.78, 2.43, and 2.71 mm for B2.7N0, B1.9N0, B1.5N0, B1.2N0, B2.7U90ST1, B1.9U90ST1, B1.5U90ST1, and B1.2U90ST1, respectively. Therefore, the B2.7U90ST1 strengthened beam showed less crack width for the same load than the other beams.

### Validation of experimental results with ACI model

The model proposed by the ACI Committee 440 (2008) is only applicable to RC beams externally reinforced with FRP materials. It is based on the classical formulation of shear strength for ordinary RC beams by adding the contribution of external shear reinforcement, that is

\[
V_u = V_c + V_S + g_f V_f
\]

with

\[
V_c = \left(1.9 \sqrt{f' c} + 2500 \phi \frac{V_{ud} d}{M_u} \right) b_w d \approx 2\sqrt{f' c} b_w d
\]

\[
V_s = \frac{A_{st} f_{yt} d}{s}
\]

\[
V_f = \frac{A_f E_f \varepsilon_{fe} d}{s_f}
\]

In the above formulas, $d$ is the effective depth of the beam section, $M_u/V_d d$ represents the shear span to depth ratio $a/d$, and $d_f = h - d'$ is the effective depth of the external reinforcement, $d'$ being the concrete cover. Furthermore, $\varepsilon_{fe}$ is the effective tensile strain in the FRP, $E_f$ is the elastic modulus of FRP in the principal fiber orientation, and $\gamma_f$ is a reduction factor equal to 0.95 for fully wrapped elements and 0.85 for beams with two or three sides bonded. In equation (4), the effective FRP strain $\varepsilon_{fe}$ is assumed to be smaller than the ultimate tensile elongation of the FRP composite $\varepsilon_{fu}$, depending on the governing mode of failure (related to the shear strengthening configuration) and can be computed as follows.
Figure 10. The normalized experimental FRP shear force with respect to ACI model.

\[
e_{f_0} = \begin{cases} 
0.004 + 0.75e_0 & \text{for fully wrapped beams} \\
0.004 & \text{for beams with two to three sides bonded} 
\end{cases}
\]

where

\[
k_v = \frac{k_1k_2L_e}{11,900e_0} \leq 0.75
\]

\[
k_1 = \left( \frac{f_c'}{27} \right) ^{2/3}
\]

\[
k_2 = \begin{cases} 
\frac{d_f-L}{d_f} & \text{for fully wrapped beams} \\
\frac{d_f-2L}{d_f} & \text{for beams with two sides bonded}
\end{cases}
\]

\[
L_e = \frac{23,300}{(4f_e)^{0.58}}
\]

For purposes of comparison, the experimental results are compared with those of the ACI model. It is clear that in the ACI model, the general design guidance is derived from the experimental data and they are only applicable to external FRP reinforcement. The results of the experimental validation are discussed below. Figure 10 shows a comparison of the experimental results with the ACI model \( V_{f,exp} / V_{f,ACI} \). The overall predictions by ACI and model also do not appear to be equally satisfactory with a mean \( V_{f,exp} / V_{f,ACI} \) value of 1.32 and a coefficient of variation (COV) of 25% for the ACI model. Therefore, the ACI model, on the other hand, provides very conservative results. It is also important to take into consideration that the ACI model is semi empirical in nature, with important governing parameters derived from test data for beams strengthened with FRP laminates, whereas the ACI model cannot be applied in certain cases.

**Conclusion**

Based on the experimental results, the following conclusions can be made:

1. The use of CFRP composites is an effective technique to enhance the shear capacity of RC beams. The externally bonded CFRP can increase the shear capacity of the beam significantly by 15%–19% than that of the control beams, depending on the variables investigated.

2. The main failure is shear failure after debonding of more than two CFRP strips due to the formulation of diagonal shear crack. Test results seem to indicate that this mechanism can be prevented by providing a larger bond length of CFRP strips in the beam from top side.

3. The results indicated that the beam depth has an influence on the angle at which primary cracking angle varied from 33, 44, 50, and 54 for beam of \( a/d \) of 2.7, 1.9, 1.5, and 1.2, respectively.

4. The impact of the beam depth is more pronounced on the ultimate load than the corresponding deflection of the control and strengthened beams.

5. The effective depth of the RC beams was increased from 185 to 417 mm which
equivalent to \(a/d\) ratio of 2.7–1.2 resulted in a decrease in average interface bond stress and an increase in the effective strain of the CFRP sheet at failure, which resulted in a higher shear capacity as compared with that of the U-wrapped beams as well as delay or mitigate the sheet debonding from the concrete surface.

6. The inclination of the primary shear crack influenced the shear strength contribution of the external strengthening. As was demonstrated in this study, the shear crack angle determined the number of CFRP strips intersected by the crack and whether or not an intersected CFRP strip was fully effective.

7. The results of the study show that the ACI model provides very conservative results with a mean of 1.32 and a COV of 25%.

Declaration of Conflicting Interests
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