Water Level Controlling System Using Pid Controller

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ABSTRACT
In certain applications such as chemical and industrial processes, it is important to keep the level of water or any other liquid in a tank or similar container at a certain desired level. In this work, we present PID based controller system where the level of water is controlled by adjusting the rate of the incoming water flow to the container by varying the speed of a DC motor water pump that is filling the container. The accuracy of the PID based controlling is demonstrated using the MATLAB simulation software.

Keywords- water level, flow rate, tank, DC motor speed, feedback control, proportional integral derivative (PID).

INTRODUCTION
Water is an essential natural resource, which is vital not only to sustain life for drinking purposes but also used in many industrial, chemical, commercial and agricultural processes. The improper use and management of water affects the sustainability of our environment. Among others in agricultural areas, water pumps need to be controlled in order to supply regulated water for the plantations or farm lands. Controlled water can be given by designing automatic electronic control systems as studied in [1], [2]. On the other hand, in chemical or industrial processes such as treatment of water in desalination plants, the level of water in a container like a tank need to be kept at certain set desired point. The level of the water need to be maintained at the desired set point for the proper functioning of the process and achieve the desired target or product. In this case, a controlling system such as proportional-integral and derivative (PID) controller plays a significant role in maintaining the accurate level by implementing the system in a feedback control system. PID controllers are used in most practical control systems ranging from consumer electronics such as cameras to industrial processes such as chemical processes [3]-[5]. The PID controller helps to get our desired output, which could be velocity in moving objects, temperature in particular environment, position at certain location, liquid level in a container and etcetera where we want to achieve a certain set point in a short possible time with minimal overshoot and with little steady state error. The performance of the PID controller are also determined by overshoots, rising time, settling time and steady state error parameters. In this work, we assumed a DC motor water pump filling a certain water container or tank and the speed of the motor determines the rate of water flow to the tank. The DC motors are used extensively in industries due to their variable speed that suits them to be used for different applications, their most demanding speed-torque characteristics and their reliability and simplicity in controlling aspects [6]. The DC motor is a highly controllable electrical actuator and is widely used for robotic manipulators, guided vehicles, steel rolling mills, cutting tool, overhead cranes, electrical traction and other applications. In comparison to AC drive, DC motor drives are simple and less expensive [6], [7]. In certain applications, the water, which could also be any other liquid in an industrial or chemical process need to be carefully supplied and kept at a certain level in the storage system. The intention of controlling the level in the storage is to achieve a perfect process control and as a result increase efficiency and productivity. A PID control theory and feedback system modelling is applied to design the overall system. As a result of maintaining the required water set level, the system not only increases process productivity but also assists towards saving and conserving water by monitoring the level and eliminating overflow in the storage system.

SYSTEM MODEL
The proposed system comprises a DC motor water pump that pumps water to the container, the PID controller and a water tank or any other storage system. Consider the diagram in Fig. 1 where the motor pumps water to the container at a rate of \( q_a \) \( [m^3/s] \) through an inlet pipe, the container with cross-sectional area \( A \) \([in^2]\), is filled to water level, \( h \) and water is leaving the container at a rate of \( q_o \) \([m^3/s]\) through an outlet pipe. Using the balance of the flows into and out of the tank, the height, \( h \) is related to \( q_{in} \) and \( q_o \) as [8]:

\[
q_{in} - q_o = A \frac{dh}{dt}
\]  

(1)

The out flow from the tank and the height can also be related assuming linear resistance to flow for simplicity of analysis and it is given as:

\[
q_o = \frac{h}{R_f}
\]

(2)

Where \( R_f \) \([in/s/m^2]\) is the flow resistance.

![Figure 1. Single Tank container filled to level h.](attachment:image.png)
The overall system is a feedback control system as shown in the block diagram given in Fig. 2. Firstly, a reference input height level, \( h_0(t) \), is set that shows the desired level the tank has to be filled. In the forward path, we have the PID controller that controls the speed of the DC motor. The speed of the motor is directly related to the water flow rate \( q_w \) supplying the tank. At the output of the overall system, we have the water level, \( h(t) \) and this information is feedback at the input and compared with the reference desired level. The error signal between the actual output and the reference, \( e(t) \) will be an input signal to the PID controller and the speed of the motor \( \omega(t) \) in rad/s will be adjusted (either increased or decreased) to control the flow rate \( q_w \) until the required target water level is achieved. The speed information of the motor is assumed to be obtained from speed measurement device such as tachometer. The speed will be transformed by the speed to height transformation (STH) block that relates the speed to the flow rate and then to the water level \( h(t) \). For simplicity purposes of the study, we assume a simple linear relationship between the speed and the incoming flow rate to the tank.

\[
\omega(t) = K_f q_w(t) \tag{3}
\]

The motor equations relating the motor speed to the physical parameters of the motor can be written as:

\[
\begin{align*}
J \ddot{\theta} + b \dot{\theta} &= T = K_i i \\
L \frac{di}{dt} + Ri &= e = V - K_e \dot{\theta}
\end{align*} \tag{4}
\]

Where \( \dot{\theta} = d\theta/dt = \omega \) is the angular speed of the motor.

MODELING THE MOTOR PUMP AND THE PID CONTROLLER

The water pump is a DC motor with the electric equivalent circuit of the armature and the free-body diagram of the rotor as shown in Fig. 3 [9]. The input to the motor is a voltage source \( V \) applied to the motor's armature, the output is the rotational speed of the shaft \( \omega(t) = d\theta/dt \). The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity. The physical parameters of the motor are \( J \): moment of inertia of the rotor \([kg.m^2]\), \( b \): motor viscous friction constant \([in\.N.m.s]\), \( K_c \): electromotive force constant \([in\.V/rad/sec]\), \( K_v \): motor torque constant \([in\.N.m/Amp]\), \( R \): electric resistance \([in\.\Omega]\), \( L \): electric inductance \([in\.H]\). Assuming armature controlled motor, the torque is proportional to the armature current \( T=K_i i \) and the back emf is proportional to the angular velocity of the shaft, \( e=K_v \dot{\theta} \). Using the above facts and based on Newton’s second law and Kirchhoff’s voltage law, the DC motor equations relating the motor speed to the physical parameters of the motor can be written as:

\[
\begin{align*}
J \ddot{\theta} + b \dot{\theta} &= T = K_i i \\
L \frac{di}{dt} + Ri &= e = V - K_e \dot{\theta}
\end{align*} \tag{5}
\]

Using the properties of Laplace transform, the above equations can be expressed in the Laplace or ‘s’ domain as [10]:

\[
s^2 J \theta(s) + s b \theta(s) = K_i I(s) \tag{6}
\]

\[
s L I(s) + R I(s) = V(s) - s K_v \theta(s) \tag{7}
\]

The angular speed in the Laplace domain is related to the angular position as \( \theta(s) = s \theta(s) \) and hence can be substituted in the above equations. By eliminating the current \( I(s) \) in (6) and (7) and after some steps, the motor rotational speed output, \( \omega(s) \), is related to the armature voltage input, \( V(s) \), as:

\[
P_w(s) = \frac{\omega(s)}{V(s)} = \frac{K_i}{s^2 J L + s (R J + b L) + Rb + K_v K_e} \tag{8}
\]

The relation in (8) represents the input–output relations of the motor-pump block shown in Fig. 2.

The PID controller is the other part of the system as shown in Fig. 2. The job of the PID controller is to adjust the output, in this case the water level at the desired set point so that in the ideal case there is no error between the sensed output, \( h(t) \), and the desired reference level. In general design of the PID, the error \( e(t) \) is related to the output \( v(t) \) as [11]:

\[
v(t) = K_p e(t) + K_i \int e(t) + K_d \frac{de(t)}{dt} \tag{9}
\]

Where \( K_p \), \( K_i \) and \( K_d \) are called the proportional gain, the integral gain and the derivative gain respectively. The proportional gain provides an overall control action proportional to the error signal, the integral gain action is to reduce steady-state errors through low-frequency compensation by an integrator and the derivative gain improves transient response through high-frequency compensation by a differentiator. By tuning the three parameters of the model, a PID controller can deal with specific process requirements. Using the Laplace domain, the input–output relation for the controller is given as:

\[
C(s) = \frac{V(s)}{e(s)} = K_p + \frac{K_i}{s} + s K_d \tag{10}
\]
OVERALL SYSTEM TRANSFER FUNCTION

As shown in Fig. 2, the overall system response is the combined response of the forward path and the feedback loop. Here the gain for the feedback loop is assumed unity and in the forward path we have the PID controller, the motor pump and the speed to height (STH) transformation block. The model for the motor pump and the PID controller is as discussed in section III. Let the impulse response of the STH system, \( g(t) \) that represents the relation between \( h(t) \) and \( \alpha(t) \). We can find an expression for \( G(s) \) the Laplace transform of \( g(t) \) by combining (1), (2) and (3). Combining (1), (2) and (3), \( \alpha(t) \) is related to \( h(t) \) as:

\[
R_f \omega(t) - K_f h = K_f R_f A \frac{dh}{dt} \quad \text{(11)}
\]

The relation in (11), in the Laplace domain is given as:

\[
R_f \omega(s) - K_f h(s) = K_f R_f A h(s) \quad \text{(12)}
\]

Therefore the transfer function \( G(s) \) becomes

\[
G(s) = \frac{h(s)}{\omega(s)} = \frac{R_f}{K_f + sK_f R_f A} \quad \text{(13)}
\]

The overall transfer function of the forward path, \( F(s) \), can be obtained by combining (8), (10) and (13)

\[
F(s) = \frac{h(s)}{e(s)} = C(s)P_m(s)G(s) \quad \text{(14)}
\]

The overall transfer function, \( G_{sys}(s) \), of the overall system including the unity feedback loop is given as:

\[
G_{sys}(s) = \frac{h(s)}{h_n(s)} = \frac{F(s)}{1 + F(s)} \quad \text{(15)}
\]

RESULTS AND DISCUSSIONS

MATLAB simulations were done to study the performance of the system and its capability to maintain and control a desired water level in the tank. Since the PID controller is integrated in MATLAB, it is easy to simulate the system and observe the results for different values of \( K_p \), \( K_i \) and \( K_d \) parameters of the controller. An optimum or acceptable result can be obtained by tuning the parameters until the design requirements are met. For this study, we assumed the following parameter values for the motor, the tank, and the STH system.

- **Motor:** \( J = 0.01 \text{ Kg.m}^2, \ b = 0.1 \text{ N.m.s}, \ K_e = 0.1 \text{ N.m/A}, \ K_n = 0.01 \text{ V/rad/s}, \ R = 1 \Omega, \ L = 0.5 \text{ H}. \)
- **Tank:** \( A=0.5 \text{ m}^2, \ R_f=0.5 \text{ s/m}^2 \)
- **STH:** \( K_f = 1 \)

Furthermore, for a unit step input reference water level for the overall system, our design criteria are to achieve an output with a settling time less than 2 seconds an overshoot less than 5% and a steady-state error less than 1%.

Fig. 4, shows the system response with and without proportional controller. As seen in the Figure, the results without the PID controller (red color) is simply the step response of the DC motor combined with the STH block. The performance without controller is very poor in terms of settling time with very high steady state error and a final value 50 % of the required reference target. It shows the requirement of a controller to obtain the desired water or liquid level. The proportional controller reduces the rise time, improves the steady state error to some extent but a ringing and an over shoot starts to appear. A proportional controller with \( K_p=10 \) looks better in terms of reducing the steady state error compared with \( K_p=5 \). For \( K_p=10 \), the settling time is about 2.95 s, an overshoot of 47%, and steady state value of 0.83. The high overshoot, settling time and steady state error necessitate the requirement of integral and derivative part to be added in the controller as not all of the design requirements can be met with a simple proportional controller.

**Figure 4.** Step response with and without proportional controller.

Fig. 5. shows the system response with the proportional-integral (PI) controller with \( K_p=K_i=10 \). The addition of the integral part to the proportional part eliminates the steady state error getting a final value near the required level compared with the Fig. 4. However, there is still a high overshoot and the settling time is also getting large.

**Figure 5.** Step response with proportional-integral (PI) controller.

The other variety of controller is the proportional-derivative (PD) controller. Fig. 6 shows the system step response with the PD controller with \( K_p=10 \) and \( K_i=5 \). It reduces the overshoot, improves the rise time and settling time but with a...
significant amount of steady state error. The final value is still not close to the desired level.

Figure 6. Step response with proportional-derivative (PD) controller

Fig. 7 shows the proportional-integral and derivative (PID) controller with \(K_p=12, K_i=15\) and \(K_d=3\). In this case the initial design specifications are met with overshoot less than 5%, settling time less than 2 seconds and steady state value of one, which is the required water level specifications. In this case, the rise time is also reduced. Therefore, with the PID controller, the desired design target is achieved. Furthermore, as shown in Fig. 8, the overshoot can be reduced at the expense of a slight increase in the settling time for the PID controller with \(K_p=9, K_i=15\) and \(K_d=2\).

Figure 7. Step response with proportional-integral derivative (PID) controller \((K_p=12, K_i=15, K_d=3)\).

Fig. 9 shows all the controllers plotted in one graph in order to have better visualization of their performances and clearly showing the super-performance of the PID controller compared with the others. With such PID controlled system, the water level can be controlled continuously without the need of manual operation, which saves time and manpower from automation. The PID controller automatically responds to the system so that the system is stabilized near the desired set point.

Figure 8. Step response with proportional-integral derivative (PID) controller \((K_p=9, K_i=15, K_d=2)\).

Table I shows a comparison of the different controllers in terms of rise time, settling time, overshoot and steady state values. From the table, we observe that a proportional controller tends to reduce the rise time and settling time but never eliminate the steady state error. There is also a large overshoot. A proportional-integral (PI) controller has the effect of eliminating the steady state error with the final value achieving the desired reference input but it makes the transient response worse with larger overshoot and also large settling time. A proportional-derivative (PD) controller has the effect of increasing the stability of the system by reducing the overshoot, the rise time, the settling time and overall improves the transient response. However, there is still large steady state error. The proportional-integral derivative (PID) controller performs well in all the parameters and satisfies the design requirements. Therefore, it is possible to fulfill the design requirements and obtain optimum performance with good accuracy by carefully tuning and selecting the proper values for \(K_p\), \(K_i\) and \(K_d\).
TABLE I. COMPARISON OF THE DIFFERENT CONTROLLERS

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>Rise time (s)</th>
<th>Settling time (s)</th>
<th>Overshoot (%)</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional (P), $K_p=10$</td>
<td>0.228</td>
<td>2.95</td>
<td>47.2</td>
<td>0.832</td>
</tr>
<tr>
<td>PI, $K_p=10$, $K_i=10$</td>
<td>0.237</td>
<td>4.64</td>
<td>49.2</td>
<td>1.0</td>
</tr>
<tr>
<td>PD, $K_p=10$, $K_d=5$</td>
<td>0.1</td>
<td>0.537</td>
<td>20.5</td>
<td>0.832</td>
</tr>
<tr>
<td>PID, $K_p=12$, $K_i=15$, $K_d=3$</td>
<td>0.17</td>
<td>0.81</td>
<td>4.45</td>
<td>1.0</td>
</tr>
<tr>
<td>PID, $K_p=9$, $K_i=15$, $K_d=2$</td>
<td>0.233</td>
<td>1.39</td>
<td>3.24</td>
<td>1.0</td>
</tr>
</tbody>
</table>

CONCLUSIONS
This paper presents a PID controlled water pump system in order to maintain a desired water level in a storage tank that is used in private, chemical, industrial or other related applications. The speed of the motor and hence the rate of water flow into the tank is controlled by adjusting the parameters of the PID controller. A step reference water level is set and the step response of the overall system is investigated in MATLAB environment in order to minimize the overshoot, improve the settling time (improved transient response) and the steady state error of the system. It is shown that suitable values for $K_p$, $K_i$, $K_d$ parameters can be found for the PID controller by tuning to maintain the desired water level for the target design specifications. The system is essential in those places where maintaining water or any other liquid level is critical for achieving the required productivity. Such controllers are powerful in controlling any similar processes that essentially require close monitoring (tight control) of the process variables or parameters that have significant impact on quality and amount of production.

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REFERENCES