FLOW OF NANOFLUID CONTAINING GYROTACTIC MICROORGANISMS OVER STATIC WEDGE IN DARCY-BRINKMAN POROUS MEDIUM WITH CONVECTIVE BOUNDARY CONDITION

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The influence of bioconvection flow of a nanofluid containing gyrotactic microorganisms over a convectively-heated wedge in a Darcy-Brinkman porous medium is analyzed numerically. The highly nonlinear governing equations using similarity transformations are developed and then computed numerically via the Keller box method. The influences of emerging parameters on fluid velocity, temperature distribution, concentration of nanoparticles, and microorganism density are presented via graphs and tables. The behavior of fluid flow is also investigated through the coefficient of skin friction, Nusselt, Sherwood numbers, and microorganism density. Results reveal that the porosity parameter reduces the boundary layers thicknesses, while the modified porosity parameter enhances the boundary layers thicknesses. With the rise of thermophoresis parameter, the thermal as well as concentration boundary layers are appreciably modulated. Motile organisms decrease with rise of Péclet number and fluid number. Finally, a comparative analysis is made through previous studies in limiting cases and shown good correlation.

KEY WORDS: nanofluid, gyrotactics microorganisms, wedge, Darcy-Brinkman, convective boundary condition

1. INTRODUCTION

The intension of boundary layer flow in a porous medium has played an important role in many engineering and scientific applications. This nature of flow is essential in a large range of technical problems, for example, the design of packed bed reactors, oil recovery techniques, environmental pollution, thermal insulation, centrifugal separation of particles, heat storage systems, and blood rheology. More applications can be seen in the books by Bejan et al. (2004), Vafai (2005), and Vadasz (2008). Generally, porous medium is significant for storage of energy and transport. The Darcy law expresses the proportionality between the pressure gradient and the velocity that has been employed to study the variety of fluids problem in saturated porous medium. This model, however, is only valid for slow flows with low permeability through porous media, whereas other porous materials such as foam metals and fibrous media usually have high porosities (Harris et al., 2009).

In porous media, inertia effects and boundary are not considered in Darcy’s model, which may change the flow characteristics with heat and mass transfer. Thus, it is very important to determine under what conditions these effects
are significant. Hong et al. (1987) have reported that, for no slip conditions, the Brinkman (1947) model, which is an extension of Darcy’s law, should be used. Ishak et al. (2008) used this model to investigate the steady flow near the stagnation point over a vertical sheet immersed in a porous medium with wall temperature. Mixed convective flow near a stagnation point towards a vertical plate immersed in a porous medium in the presence of variable heat flux was scrutinized by Rosali et al. (2011). Pantokratoras (2015) studied forced convective flow with heat transfer past a plane sheet in the presence of convective boundary condition in a Darcy-Brinkman porous medium. Tian et al. (2016) analyzed the effect of viscous dissipation on flow of a power law fluid in a flat channel immersed in a Darcy-Brinkman-Forchheimer porous medium. Kumar and Sood (2016) explored the electrically conducting fluid on mixed convective flow near a stagnation-point towards a shrinking sheet embedded in a porous medium. Sheikholeslami (2017a) investigated the impact of magnetic field on free convection flow of a nanofluid in an open porous cavity using the Lattice Boltzmann method. Sheikholeslami (2017b) also studied the impact of Lorentz forces on natural convection flow of a nanofluid embedded in porous cylinder with Darcy and KKL models. Free convection flow containing CuO-water based nanofluid in a porous cavity by taking Darcy law was scrutinized by Sheikholeslami (2017c). Sheikholeslami and Shehzad (2017a) studied MHD water based CuO nanofluid in a porous semi-annulus in the presence of constant heat flux using KKL and Darcy models. Sheikholeslami (2017d) obtained the numerical solution of natural convective flow in a porous curved cavity in the presence of external magnetic source with nanofluid. Ali et al. (2017) studied the influence of slip parameters in the MHD flow of a non-Newtonian viscoelastic fluid past a permeable oscillatory stretching surface embedded in porous medium.

The viscous flow over a wedge has been extensively examined by many researchers due to applications in engineering such as heat exchanger, extraction of crude oil, thermal insulation, water pollution ground, and nuclear waste storage. Falkner and Skan (1931) initially pursued the two-dimensional viscous flow past a wedge. Riley and Weidman (1989) obtained the dual solutions of Falkner and Skan over a stretched boundary. The flow over a moving wedge in the presence of suction/injection was explored by Ishak et al. (2007). Jafar et al. (2013) scrutinized MHD flow over a moving wedge with parallel free stream. Ali and Khan (2016) discussed the influence of heat transfer on generalized Carreau fluid over both static and moving wedges. Ganapathirao et al. (2016) discussed the combined effects of suction/injection MHD mixed convection flow past a vertical wedge immersed in a porous medium.

Enhancement of heat transfer is a major concern in engineering and industrial processes. Various techniques have been used to enhance the efficiency of heat transfer such as changing geometry of flow, boundary conditions, or by increasing the fluids thermal conductivity. One technique is the suspension of micro-sized ultrafine solid particles in base fluid referred as Nanofluid. Nanofluid is a fluid that is formed by scattering nanometer sized solid particles and/or fibers having diameters less than 100 nm. Regular heat transfer fluids (i.e., water, bio fluids, engine oil, and ethylene glycol) have lower thermal conductivities that cannot congregate with the requirement of modern technologies of cooling. Choi and Eastman (1995) dispersed the particles made of metal or metal oxide into regular fluids to improve the thermal conductivity.

Buongiorno (2006) proposed a model of convective transport for nanofluid. He observed that the Brownian motion parameter and thermophoresis diffusion effect of nanoparticles give massive enhancement in the thermal conductivity of fluid. Nield and Kuznetsov (2009, 2010) initially examined the boundary layer flow of a nanofluid along a vertical surface. Khan and Pop (2010) extended the work of Nield and Kuznetsov by considering a constant surface temperature over a stretching surface. The impacts of thermophoresis and Brownian motion of a nanofluid past a vertical plate with properties of variable fluid was studied by Afify and Bazid (2014). Kandelouisi (2014) discussed the impact of external variable magnetic field on ferrofluid flow with heat transfer at uniform heat flux. Sheikholeslami (2014) discussed the behavior of hydrothermal on nanofluid between two parallel plates, and the influence of uniform suction on boundary layer flow and heat transfer of a nanofluid past a cylinder (Sheikholeslami, 2015). Freidoonimehr et al. (2015) studied MHD unsteady free convection flow of four different water-based nanofluids towards a porous vertical stretching surface. Yasin et al. (2016) analyzed the steady flow of double-diffusive mixed convective nanofluid over a vertical flat plate immersed in a porous medium. Afify and Elgazery (2016) explored the influence of electrically conducting Maxwell fluid past a stretching surface with chemical reaction and nanoparticles. Sheikholeslami studied the influence of magnetic source containing water based Fe$_3$O$_4$ nanofluid in a cavity with circular hot cylinder (2016a), the hydrothermal behavior on nanofluid in a porous curved enclosure in the presence of a magnetic field (2016b), and the characteristics of Lorentz forces on forced convection flow of a nanofluid using Koo-Kleinstreuer-Li
model (Sheikholeslami et al., 2017a). Sheikholeslami and Rokni (2017a) observed the influence of buoyancy ratio on induced magnetic field holding nanofluid with suction. Sheikholeslami and Chamkha (2017) investigated the impact of MHD forced convective flow of a nanofluid with Marangoni convection in a two phase model. Sheikholeslami and Shehzad (2017b) examined the effect of thermal radiation on nanofluid in the presence of Lorentz forces with variable viscosity. Reddy and Chamkha (2017) studied the characteristics of heat and mass transfer of water-based alumina and silver nanofluid past a vertical cone through porous media in the presence of heat generation/absorption. Recently, various aspects of heat transfer characteristics on nanofluid under various conditions have been investigated (Sheikholeslami, 2017e,f,g; Sheikholeslami et al., 2017b; Sheikholeslami and Vajravelu, 2017; Sheikholeslami and Rokni, 2017b; Sheikholeslami and Bhatti, 2017a,b).

The patterns of bioconvection flow in bacteria have a larger density compared to water in fluid at the boundary. Because of this unstable position, the bacterial boundary layer is scattered into bioconvection cells. There are two types of microorganisms, gyrotactic and oxytoctic. The fundamental mechanism of bioconvection is invariant for both models. Thus, there has been more research on boosting the rate of mass transfer and species of concentration due to industrial applications such as chemical processing, extrusion process, process of conveyor belt, technology of surface charge, and biotechnological applications. Kuznetsov and Avramenko (2004) and Geng and Kuznetsov (2004) were probably the first to consider the bioconvection flow containing gyrotactic microorganisms. Kuznetsov (2011) studied the free convective flow of nanofluid containing microorganism density and nanoparticles. Free convective flow in water-based nanofluid involving microorganisms was examined by Aziz et al. (2012). Khan et al. (2014) studied the MHD flow containing nanoparticles and gyrotactic microorganisms past a plate with Navier slip. Mutuku and Makinde (2014) investigated the buoyancy forces on hydromagnetic flow over a vertical moving plate containing nanoparticles and gyrotactic microorganisms. Sarkar et al. (2016) explored the impact of bioconvection flow of nanofluid holding nanoparticles and microorganisms towards a stretching sheet embedded in a non-Darcy porous medium.

In this article, we report our preliminary study of the impact of convective boundary condition on flow of a nanofluid holding gyrotactic microorganisms over a static wedge in a Darcy-Brinkman porous medium. Similarity equations are developed and a numerical solution was obtained via Keller-box method. Graphs and tables illustrate the influences of the physical parameters.

2. PROBLEM FORMULATION

Consider a steady two-dimensional incompressible flow of a nanofluid containing gyrotactic microorganisms over a static wedge immersed in a Darcy-Brinkman porous medium with convective boundary condition (Fig. 1). The external velocity from the wedge is \( \bar{u}_c(\bar{x}) = c\bar{x}^n \), where \( c \) and \( n \) are positive constants. It is assumed that the lower surface of the wedge was heated convectively with temperature \( \bar{T}_f \) which gives a coefficient of heat transfer \( \bar{h}_f \).

FIG. 1: Flow diagram of the problem
\( T_w, C_w, \) and \( \tilde{C}_w \) are the convective temperature, concentration of nanoparticle, and motile microorganism density at the wedge, respectively, and are taken to be higher than \( T_\infty, C_\infty, \) and \( G_\infty \) (i.e., ambient temperature, ambient concentration of nanoparticle, and ambient motile microorganism density, respectively). The governing equations under these assumptions with the usual boundary layer are written as:

\[
\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{1}
\]

\[
\frac{\partial \tilde{T}}{\partial x} + \frac{\partial \tilde{T}}{\partial y} = \tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} = \frac{\partial^2 \tilde{T}}{\partial y^2} + \frac{\tilde{C}}{K_1} \left( \tilde{u} - \tilde{u}_e \right) \tag{2}
\]

\[
\frac{\partial \tilde{C}}{\partial x} + \frac{\partial \tilde{C}}{\partial y} = \frac{\partial^2 \tilde{C}}{\partial y^2} + \frac{\partial \tilde{T}}{\partial y} \frac{\partial \tilde{T}}{\partial y} \left( \frac{\partial^2 \tilde{T}}{\partial y^2} \right)^2 \tag{3}
\]

\[
\frac{\partial \tilde{G}}{\partial x} + \frac{\partial \tilde{G}}{\partial y} = \frac{\partial^2 \tilde{G}}{\partial y^2} + \frac{\tilde{b} \tilde{w}_e}{(C_w - C_\infty)} \frac{\partial \tilde{G}}{\partial y} \frac{\partial \tilde{T}}{\partial y} + \frac{\tilde{C}}{n} \frac{\partial^2 \tilde{C}}{\partial y^2} \tag{4}
\]

The physical boundary conditions are:

\[
\tilde{u} = 0, \hspace{1em} \tilde{v} = 0, \hspace{1em} k \frac{\partial \tilde{T}}{\partial y} = k_f \left( \tilde{T}_f - \tilde{T} \right), \hspace{1em} \tilde{C} = \tilde{C}_w, \hspace{1em} \tilde{G} = \tilde{G}_w \hspace{1em} \text{at} \hspace{1em} \tilde{y} = 0,
\]

\[
\tilde{u} = \tilde{u}_e(x), \hspace{1em} \tilde{T} \rightarrow T_\infty, \hspace{1em} \tilde{C} \rightarrow \tilde{C}_\infty, \hspace{1em} \tilde{G} \rightarrow \tilde{G}_\infty \hspace{1em} \text{as} \hspace{1em} \tilde{y} \rightarrow \infty,
\]

where \( \tilde{u} \) and \( \tilde{v} \) are the velocity components in the \( \tilde{x} - \) and \( \tilde{y} - \) axes, respectively, \( \tilde{v}_{eff} = \tilde{u}_{eff} / \tilde{p} \) is the effective kinematic viscosity, \( \tilde{u}_{eff} \) is the effective (or “apparent”) viscosity, \( \tilde{p} \) is the density, \( \tilde{v} \) is the normal kinematic viscosity, \( \tilde{C} \) is the porosity parameter of the porous medium, \( K_1 \) is the permeability of the porous medium, \( \tilde{\alpha} \) is the thermal diffusivity, \( \tilde{T} \) is the temperature, \( \tilde{C} \) is the concentration of nanoparticle, \( \tilde{G} \) is the motile organism density, \( \tilde{D}_B \) and \( \tilde{D}_T \) are the coefficients of Brownian and thermophoresis diffusion, respectively, \( \tau \) is the ratio between the effective heat capacity of the nanoparticle material and specific heat capacitance of the fluid, \( \tilde{D}_n \) is the microorganism diffusion coefficient, \( b \) is the chemo taxis constant, and \( \tilde{w}_e \) is the maximum swimming cell speed.

Now, we introduce the similarity transformation (Khan et al., 2012):

\[
\eta = \tilde{y} \sqrt{\frac{c(n + 1)}{2\tilde{x}}} \tilde{x}^{-n/2}, \hspace{1em} \psi = \sqrt{2\tilde{\alpha}} \tilde{x}^{n/2} f(\eta), \hspace{1em} \theta(\eta) = \frac{\tilde{T} - T_\infty}{\tilde{T}_f - T_\infty}, \hspace{1em} \varphi(\eta) = \frac{\tilde{C} - C_\infty}{C_w - C_\infty}, \hspace{1em} \chi(\eta) = \frac{\tilde{G} - G_\infty}{G_w - G_\infty}, \hspace{1em} \tilde{h}_f = k c(n + 1) \tilde{x}^{-n/2} \frac{2\tilde{\alpha}}{2\tilde{x}}, \hspace{1em} K_1 = \frac{2n^{1 - n}}{n + 1} \tag{7}
\]

Here \( \eta \) is the similarity variable, \( \psi \) is the stream function.

In view of relation (7), Eqs. (2)–(6) are transmuted to:

\[
\Lambda f'''' + f f'' + \beta \left( 1 - f''^2 \right) - K \left( f' - 1 \right) = 0, \tag{8}
\]

\[
\theta'' + f \theta' + N b \theta' \varphi' + N t \theta'' = 0, \tag{9}
\]

\[
\varphi'' + S c f \varphi' + \frac{N t}{N b} \theta'' = 0, \tag{10}
\]

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\[ \chi'' + Sb f \chi' - Pe \varphi' \chi' - Pe \left( \Omega + \chi \right) \varphi'' = 0. \]  
(11)

The converted boundary conditions are:

\[
\begin{align*}
    f(0) &= 0, & f'(0) &= 0, & \varphi'(0) &= -\gamma (1 - \theta(0)), & \varphi(0) &= 1, & \chi(0) &= 1, \\
    f'(\infty) &\to 1, & \theta(\infty) &\to 0, & \varphi(\infty) &\to 0, & \chi(\infty) &\to 0,
\end{align*}
\]  
(12)

where prime indicate the differentiation to \( \eta \). \( \Lambda = \tilde{\chi}^2 \text{Pr} = \frac{\tilde{\chi}^2 \tilde{\nu}_{eff}}{\tilde{\alpha}} \) is the modified permeability parameter, \( \beta = (2n)/(n + 1) \) is the Hartree pressure gradient parameter, \( K = \frac{\tilde{\chi}^2 \tilde{\nu}}{c} \) is the porosity parameter, \( Nb = \tilde{\tau} \tilde{D}_B \left( \tilde{C}_w - \tilde{C}_\infty \right) / \tilde{\alpha} \) is the Brownian motion parameter, \( Nt = \tilde{\tau} \tilde{D}_f \left( \tilde{T}_f - \tilde{T}_\infty \right) / \tilde{T}_\infty \tilde{\alpha} \) is the thermophoresis parameter, \( Sc = \tilde{\alpha} / \tilde{D}_B \) is the Schmidt number, \( \text{Pe} = \tilde{b} \tilde{\omega}_c / \tilde{D}_n \) is the Péclet number, \( \Omega = \tilde{G}_\infty / \left( \tilde{G}_w - \tilde{G}_\infty \right) \) is the fluid number, and \( Sb = \tilde{\alpha} / \tilde{D}_n \) is the bio-convection Schmidt number. It is worth mentioning that the value of \( n = 0 (\beta = 0) \) represents the flow over a horizontal plate, whereas \( n = 1 (\beta = 1) \) implies flow over a vertical plate.

The significant physical quantities are the local skin friction coefficient, the local Nusselt number, local Sherwood number, and microorganism density number, which are defined as:

\[
C_{fx} = \frac{\tilde{x}_w}{\rho \tilde{u}_c^2}, \quad \text{Nu}_x = \frac{-\tilde{x} q_w}{k (\tilde{T}_f - \tilde{T}_\infty)}, \quad \text{Sh}_x = \frac{\tilde{x} m_w}{\tilde{D}_B (\tilde{C}_w - \tilde{C}_\infty)}, \quad \text{Sb}_x = \frac{\tilde{x} k_w}{\tilde{D}_n (\tilde{G}_w - \tilde{G}_\infty)},
\]  
(13)

where \( \tilde{x}_w \) is the shear stress, \( q_w \) is the heat flux, \( m_w \) is the mass flux, and \( k_w \) is motile microorganisms flux described as:

\[
\tilde{x}_w = \mu \left( \frac{\partial \tilde{u}}{\partial \tilde{y}} \right)_{\tilde{y}=0}, \quad q_w = -k \left( \frac{\partial \tilde{T}}{\partial \tilde{y}} \right)_{\tilde{y}=0}, \quad m_w = -\tilde{D}_B \left( \frac{\partial \tilde{C}}{\partial \tilde{y}} \right)_{\tilde{y}=0}, \quad k_w = -\tilde{D}_n \left( \frac{\partial \tilde{G}}{\partial \tilde{y}} \right)_{\tilde{y}=0}.
\]  
(14)

That is,

\[
C_{f} \text{Re}_x^{1/2} / \text{Pr}^{1/2} = \sqrt{\frac{n + 1}{2} f''(0)}, \quad \text{Nu}_x \text{Pe}_x^{-1/2} = -\sqrt{\frac{n + 1}{2}} \varphi'(0),
\]  
(15)

\[
\text{Sh}_x \text{Pe}_x^{-1/2} = -\sqrt{\frac{n + 1}{2}} \varphi'(0), \quad \text{Sb}_x \text{Pe}_x^{-1/2} = -\sqrt{\frac{n + 1}{2}} \chi'(0),
\]

where \( \text{Re}_x = \tilde{x} u_c / \tilde{\nu} \) is the local Reynolds number, and \( \text{Pe}_x = \tilde{x} u_c / \tilde{\alpha} \) is the Péclet number.

**3. SOLUTION PROCEDURE**

In the current study, a useful implicit finite difference scheme, the Keller box method, has been utilized to observe the flow problem described by the transformed Eqs. (8)–(11). This method is widely used to solve parabolic equations, as described by Cebeci and Bradshaw (1988), and is summarized in the following steps:

1. Using new dependent variables to transform the Eqs. (8)–(11) to a first order system.
2. Using central differences to write the difference equation.
3. Using Newton’s method to linearize the transformed algebraic equations and then write these equations in matrix vector form.
4. Using a block tri-diagonal elimination method to solve the obtained linear system.

The step size is taken as \( \Delta \eta = 0.01 \) and the corrected results are obtained accurately to \( 10^{-5} \), which fulfills the criterion of convergence. The satisfied outer boundary condition is obtained by taking the thickness of boundary layer \( \eta_{\infty} = 10 \).
4. RESULTS AND DISCUSSION

The impacts of pertinent parameters involving in fluid flow problem are discussed through graphs and tables. Table 1 compares our results of $f''(0)$ with those of Yih (1998) and Yacob et al. (2011) in limiting cases that are matched closely, which assured the validity of the current methodology. Table 2 demonstrates the influence of porosity parameter $K$ against modified porosity parameter $\Lambda$ on $C_f \text{Re}^{1/2}_x$, $\text{Nu}_x \text{Re}^{-1/2}_x$, $\text{Sh}_x \text{Re}^{-1/2}_x$ and $Sb_x \text{Re}^{-1/2}_x$ for $\beta = 0$ and $\beta = 1$. It is clear that the values of $C_f \text{Re}^{1/2}_x$, $\text{Nu}_x \text{Re}^{-1/2}_x$, $\text{Sh}_x \text{Re}^{-1/2}_x$ and $Sb_x \text{Re}^{-1/2}_x$ decrease with increasing $\Lambda$ for $\beta = 0$ and $\beta = 1$, while the values are significantly enhanced with larger values of $K$. Moreover, the skin friction, the Nusselt, the Sherwood numbers, and motile microorganism number are positive, indicating that fluid exerts a drag force on the wedge, whereas heat and mass species is moved from the hot surface to the cold fluid.

**TABLE 1:** Comparison of $f''(0)$ for different $n$ when $K = 0$, $\Lambda = 1$, $\beta = 2n/(1+n)$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Yih (1998)</th>
<th>Yacob et al. (2011)</th>
<th>Present</th>
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<tbody>
<tr>
<td>0</td>
<td>0.469600</td>
<td>0.4696</td>
<td>0.4696</td>
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<td>0.9277</td>
<td>0.9277</td>
</tr>
<tr>
<td>0.5</td>
<td>—</td>
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<td>1.0389</td>
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</table>

**TABLE 2:** Values of skin friction, Nusselt number, Sherwood number, and microorganism density number versus $K$ for different values of $\Lambda$ when $Nt = 0.1$, $Sc = Sb = 1$, $\gamma = 0.3$, $Pe = \Omega = 0.1$ are fixed

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\Lambda$</th>
<th>$C_f \text{Re}^{1/2}_x$</th>
<th>$\text{Nu}_x \text{Re}^{-1/2}_x$</th>
<th>$\text{Sh}_x \text{Re}^{-1/2}_x$</th>
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</tbody>
</table>
Figures 2–5 present the impact of porosity parameter $K$ on the velocity, temperature distribution, concentration of nanoparticles, and microorganisms density profile for $\beta = 0$ and $\beta = 1$. As the velocity distribution increases with larger values of $K$ for both $\beta$, the velocity boundary layer decreases (Fig. 2). Physically, as the porous parameter increases, the regime becomes more porous, and as a result the Darcian force moderates in magnitude. This resistance of the Darcian force decelerates the fluids particles. This reduces the resistance as the porous parameter increases. Gradually the flow experiences a less drag and thereby decreases the flow retardation. Hence the porous parameter boosts the fluid motion within the boundary layer.

The temperature, concentration of nanoparticles, and microorganism density decrease with increasing $K$ for $\beta = 0$ and $\beta = 1$, as shown in Figs. 3–5. Moreover, the thicknesses of boundary layers are larger for flow over a horizontal plate compared to flow over a vertical plate. The influence of modified permeability parameter $K$ on velocity, temperature distribution, concentration of nanoparticles, and microorganism density for $\beta = 0$ and $\beta = 1$ are depicted in Figs. 6–9. The velocity of fluid clearly decreases with higher values of $\Lambda$ for $\beta = 0$ and $\beta = 1$ (Fig. 6). Thus the velocity boundary layer becomes larger for both cases. The temperature distribution, nanoparticle concentration, and microorganism density are significantly enhanced with increasing $\Lambda$ for $\beta = 0$ and $\beta = 1$ (Figs. 7–9).

**FIG. 2:** The velocity profile for different values of $K$ when $\Lambda = 3$

**FIG. 3:** The temperature profile for different values of $K$ when $\Lambda = 3$, $Nb = Nt = 0.1$, $\gamma = 0.3$
FIG. 4: The concentration profile for different values of $K$ when $\Lambda = 3$, $Nb = Nt = 0.1$, $\gamma = 0.3$, $Sc = 1$

FIG. 5: The microorganisms density profile for different values of $K$ when $\Lambda = 3$, $Nb = Nt = 0.1$, $\gamma = 0.3$, $Pe = \Omega = 0.1$, $Sc = Sb = 1$

FIG. 6: The velocity profile for different values of $\Lambda$ when $K = 0.5$
FIG. 7: The temperature profile for different values of $\Lambda$ when $K = 0.5, Nb = Nt = 0.1, \gamma = 0.3$

FIG. 8: The concentration profile for different values of $\Lambda$ when $K = 0.5, Nb = Nt = 0.1, \gamma = 0.3, Sc = 1$

FIG. 9: The microorganisms density profile for different values of $\Lambda$ when $K = 0.5, Nb = Nt = 0.1, \gamma = 0.3, Pe = \Omega = 0.1, Sc = Sb = 1$
The effect of Brownian motion $Nb$ on the temperature distribution and concentration of nanoparticles for $\beta = 0$ and $\beta = 1$ is seen in Figs. 10 and 11. The temperature distribution and thermal boundary layer increase with enhanced $Nb$ (Fig. 10), because the kinetic energy of the nanoparticles increases due to the strength of this chaotic motion and as a result, the fluid temperatures increase. Figure 11 reveals that the concentration of nanoparticles decreases due to increasing values of $Nb$. It can be concluded that the Brownian motion parameter makes the fluid warm within the boundary, and at that time aggravates deposition particles away from the regime of fluid to the surface that causes a decrease in concentration of nanoparticles as well as the thickness of concentration boundary layer. The larger values of Brownian motion imply a strong behavior for the smaller particles, whereas smaller values of $Nb$ applied for stronger particles.

The influence of the thermophoresis parameter $Nt$ on the temperature distribution and concentration of nanoparticles are shown in Figs. 12 and 13, which reveal that for larger values of $Nt$, the temperature distribution and concentration of nanoparticles increase for flow over a flat plate, as well as for stagnation point flow. This is because diffusion penetrates deeper into the fluid due to increasing values of $Nt$, which causes thickening of both the thermal boundary layer and the concentration boundary layer. It is interesting to note that the effect of thermophoresis is more pronounced on the concentration of nanoparticles when compared to temperature distribution.

**FIG. 10:** The temperature profile for different values of $Nb$ when $K = 0.5$, $\Lambda = 2$, $Nt = 0.1$, $\gamma = 0.3$

**FIG. 11:** The concentration profile for different values of $Nb$ when $K = 0.5$, $\Lambda = 2$, $Nt = 0.1$, $\gamma = 0.3$, $Sc = 1$
The Hartree pressure gradient parameter $\beta$ affects the velocity, temperature distribution, concentration of nanoparticles, and microorganism density, as seen in Figs. 14–17. Increasing $\beta$ results in an accelerated velocity distribution, leading to a thinner velocity boundary layer (Fig. 14). Physically, $\beta$ reflects the pressure gradient, with positive values favoring a favorable pressure gradient that enhances flow within the boundary. The velocity profile crush occurs nearer and closer to the wedge without reverse flow.

The temperature distribution, concentration of nanoparticles, and microorganism density decrease upon increasing $\beta$, as shown in Figs. 15–17. The convective parameter $\gamma$ influences the temperature distribution and concentration of nanoparticles as depicted in Figs. 18 and 19. Higher $\gamma$ values, indicative of powerful convective heating at the wedge surface, increase the temperature and concentration at the wedge surface. This permits a deeper thermal effect, as well as species, to penetrate the quiescent fluid. Consequently, the temperature and concentration, as well as the thermal and concentration boundary layer thicknesses, grow with increasing $\gamma$.

The microorganism density for various $Péclet$ number $Pe$ and fluid number $\Omega$ is illustrated in Figs. 20 and 21. It is evident that for flow over horizontal and vertical plates, the microorganism density decreases with higher $Pe$ due to the ratio of unbalancing times scale. The $Péclet$ number is the ratio of cell swimming speed to microorganism diffusivity. With increased $Pe$, the microorganisms' density decreases, leading to enhanced speed of cell swimming, while...
FIG. 14: The velocity profile for different values of $\beta$ when $\Lambda = 3$, $K = 0.5$

FIG. 15: The temperature profile for different values of $\beta$ when $K = 0.5$, $\Lambda = 3$, $Nt = Nb = 0.1$, $\gamma = 0.3$

FIG. 16: The concentration profile for different values of $\beta$ when $K = 0.5$, $\Lambda = 3$, $Nt = Nb = 0.1$, $\gamma = 0.3$, $Sc = 1$
FIG. 17: The microorganisms density profile for different values of $\beta$ when $K = 0.5$, $\Lambda = 3$, $Nb = Nt = 0.1$, $\gamma = 0.3$, $Pe = \Omega = 0.1$, $Sc = Sb = 1$

FIG. 18: The temperature profile for different values of $\gamma$ when $K = 0.5$, $\Lambda = 3$, $Nb = Nt = 0.1$, $\beta = 1$

FIG. 19: The concentration profile for different values of $\gamma$ when $K = 0.5$, $\Lambda = 3$, $Nb = Nt = 0.1$, $\beta = 1$, $Sc = 1$
increases in bioconvection that ultimately reduces the microorganism density. A similar trend is also observed for fluid parameter as shown in Fig. 21. Due to high pressure force, the microorganism density profile are lower for flow over a horizontal plate compared to flow over a vertical plate.

Finally, for the verification of the accuracy of the applied numerical method, we compared our results corresponding to the concentration of nanoparticle profile with the available published results of Khan and Pop (2013) in Fig. 22, which are found to be in excellent agreement.

5. CONCLUSION

The combined effects of microorganisms and nanoparticles on boundary layer flow past a static wedge embedded in Darcy-Brinkman porous medium with convective boundary condition were explored numerically. The governing equations were developed into ordinary differential equations using similarity transformations and then solved numerically via the Keller-box method. The following interesting results can be summarized from this research for $\beta = 0$ and $\beta = 1$:
1. The velocity of fluid increases due to the porosity parameter, whereas the velocity decreases due to a modified permeability parameter. Temperature of fluid and concentration profile decrease due to porosity parameter and increase as modified parameter increases.

2. Impacts of thermophoresis and Brownian motion on the temperature distribution are similar.

3. Influences of thermophoresis and Brownian motion on the concentration of nanoparticles are opposite.

4. Impacts of Péclet and fluid parameters on the microorganism density are attenuated for flows over vertical and horizontal plates.

5. Hartree pressure gradient accelerates the velocity profile and decelerates the temperature, concentration, and microorganisms.

6. The skin friction, Nusselt, Sherwood numbers, and motile microorganism density significantly increase due to porosity but decrease due to modified porosity.

REFERENCES


Gyrotactic Microorganisms in Darcy-Brinkman Porous Medium


