Performance evaluation of irreversible Miller engine under various specific heat models

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Abstract

In this paper, the performance of a Miller engine is evaluated under different specific heat models (i.e., constant, linear, and fourth order polynomial). Finite-time thermodynamics is used to derive the relations between power output and thermal efficiency at different compression and expansion ratios for an ideal naturally-aspirated (air-standard) Miller cycle. The effect of the temperature-dependent specific heat of the working fluid on the irreversible cycle performance is significant. It was found that an accurate model such as fourth order polynomial is essential for accurate prediction of cycle performance. The conclusions of this investigation are of importance when considering the designs of actual Miller engines.

Keywords: Finite-time thermodynamics; Miller cycle; Friction; Temperature-dependent specific heat

1. Introduction

The Miller cycle has an expansion ratio exceeding its compression ratio. The Miller cycle, shown in Fig. 1, is a modern modification of the Atkinson cycle (i.e., a complete expansion cycle). The compression ratio of spark-ignition, gasoline-fuelled engines is limited by knock and fuel quality to be in the range between 8 and 11, depending upon various factors, such as the engine’s bore and stroke as well as engine speed. Significant achievements have arisen since finite-time thermodynamics was developed in order to analyze and optimize the performances of real heat-engines [1–3]. Finite-time thermodynamics was used to derive the relations between thermal efficiency, compression and expansion ratios for an ideal naturally-aspirated (air-standard) Miller cycle assuming that the specific heat of the working fluid varies linearly with temperature [4]. They concluded that the effect of the temperature-dependent specific heat of the working fluid on the irreversible cycle performance is significant and depends on the slope and the interception of the linear equation.

Mathematical techniques were developed for optimal-control theory, to reveal the optimal motions of the pistons in Diesel [5] and Otto cycle engines [6]. The performance of internal-combustion engine cycles was evaluated using the optimal motion of a piston fitted in a cylinder containing a gas pumped at a specified heating-rate [7,8]. The power and efficiency upper limits for internal-combustion engines were deduced [9]. The behaviours of Otto, Diesel and Dual

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cycles with friction losses over a finite period were modelled [10–12]. The effects of heat transfers on the performances of Otto and Diesel cycles were investigated [13]. The relations between the net power and the efficiency for Diesel, Otto and Dual cycles with consideration of heat-transfer losses were derived [14–16]. The characteristics of power and efficiency for Otto and Dual cycles with heat transfers and friction losses were determined [17,18] and the optimal power-density characteristics for Atkinson, Miller and Dual cycles without any such losses studied [19–21]. The universal power and efficiency characteristics for irreversible reciprocating heat-engine cycles with heat transfers and friction losses were reported [22]. The performance of an irreversible Dual cycle was optimized [23]. The predicted behaviour was corroborated by experimental results. A quantitative simulation of an Otto engine’s behaviour can be accurately achieved by a simple Novikov model with heat leaks that was found [24]. Recently, it was found that friction and the temperature-dependent specific heat of the working fluid of a Diesel engine had significant influences on its power output and efficiency [25]. Also, a study on the performance of irreversible air-standard Miller cycle using constant specific heat was presented [26]. This paper describes a corresponding analysis of the behaviour for an irreversible Miller cycle with losses arising from heat resistance and friction, by using various specific heat models. The
A fourth order polynomial model for the specific heats used in this work has never been used in such a study to the best of the authors’ knowledge.

2. An air-standard Miller cycle model

The Miller cycle under consideration is shown in Fig. 1, the compression $1 \rightarrow 2$ process is isentropic; the heat addition $2 \rightarrow 3$ an isochoric process; the expansion $3 \rightarrow 4$ an isentropic process; and the heat rejection $4 \rightarrow 5$ an isochoric process, while the rejection of heat $5 \rightarrow 1$ an isobaric process. Finally the exhaust from $1 \rightarrow 6$ is also an isobaric process. As is usual in finite-time thermodynamic heat-engine cycle models, there are two instantaneous adiabatic processes namely $1 \rightarrow 2$ and $3 \rightarrow 4$. For the heat addition $2 \rightarrow 3$ and heat rejection $4 \rightarrow 5$ stages respectively, it is assumed that heating occurs from state 2 to state 3 and cooling ensues from state 4 to state 1. These processes proceed according to:

$$\frac{dT}{dt} = \frac{1}{C_1} (\text{for } 2 \rightarrow 3); \quad \frac{dT}{dt} = \frac{1}{C_2} (\text{for } 4 \rightarrow 5); \quad \frac{dT}{dt} = \frac{1}{C_3} (\text{for } 5 \rightarrow 1)$$

where $T$ is the absolute temperature and $t$ is the time, $C_1$, $C_2$ and $C_3$ are constants. Integrating the equations in Eq. (1) yields

$$t_1 = C_1(T_3 - T_2); \quad t_2 = C_2(T_4 - T_3); \quad \text{and } t_3 = C_3(T_3 - T_1) \quad (2)$$

where $t_1$ is the heating period and $t_2$ and $t_3$ the cooling periods. Then, the cycle period is

$$\tau = t_1 + t_2 + t_3 = C_1(T_3 - T_2) + C_2(T_4 - T_3) + C_3(T_3 - T_1) \quad (3)$$

In a real cycle, the specific heat of the working fluid depends upon its temperature and this will influence the performance of the cycle. Over the temperature range from 200 to 6000 K, the specific heat curve is assumed to follow a fourth order polynomial, introduced by a team of researches called the NASA polynomial equation [27]. The results of the fourth order polynomial will be compared with the linear model used by Al-Sarkhi et al. [4] and the constant value of specific heat independent of temperature. Fig. 2 shows the different specific heat models that are available in the literature and applied in the present study on the Miller cycle.

For the temperature range 200–1000 K,

$$\frac{C_p}{R} = 3.56839 - 6.78729 \times 10^{-4}T + 1.5537 \times 10^{-6}T^2 - 3.29937 \times 10^{-12}T^3 - 4.66395 \times 10^{-13}T^4 \quad (4)$$

and for the temperature range 1000–6000 K,

$$\frac{C_p}{R} = 3.08793 + 1.24597 \times 10^{-3}T - 4.23719 \times 10^{-7}T^2 + 6.74775 \times 10^{-11}T^3 - 3.97077 \times 10^{-15}T^4 \quad (5)$$
$C_p$ and $C_v$ are the molar specific heats with respect to constant pressure and volume, respectively. They are related as:

$$C_v = C_p - R.$$ 

Accordingly, $R$ is the working fluid constant.

The heat added to the working fluid, during process 2→3, is

$$Q_{in} = M \int_{T_2}^{T_3} C_v dT = M \int_{T_2}^{T_3} R((a_1 - 1) + a_2 T + a_3 T^2 + a_4 T^3 + a_5 T^4) dT$$

which by integration yields

$$Q_{in} = MR \left[ (a_1 - 1)(T_3 - T_2) + \frac{a_2}{2} (T_3^2 - T_2^2) + \frac{a_3}{3} (T_3^3 - T_2^3) + \frac{a_4}{4} (T_3^4 - T_2^4) + \frac{a_5}{5} (T_3^5 - T_2^5) \right]$$

Where $M$ is the molar number of the working fluid. Heat rejected by the working fluid, during process 4→5, is

$$Q_{out-1} = M \int_{T_5}^{T_4} C_p dT = M \int_{T_5}^{T_4} R((a_1 - 1) + a_2 T + a_3 T^2 + a_4 T^3 + a_5 T^4) dT$$

Eq. (8) results in

$$Q_{out-1} = MR \left[ (a_1 - 1)(T_4 - T_5) + \frac{a_2}{2} (T_4^2 - T_5^2) + \frac{a_3}{3} (T_4^3 - T_5^3) + \frac{a_4}{4} (T_4^4 - T_5^4) + \frac{a_5}{5} (T_4^5 - T_5^5) \right]$$

The heat rejected by the working fluid, during process 5→1, is

$$Q_{out-2} = M \int_{T_1}^{T_5} C_p dT = M \int_{T_1}^{T_5} R(a_1 + a_2 T + a_3 T^2 + a_4 T^3 + a_5 T^4) dT$$

Eq. (10) yields

$$Q_{out-2} = MR \left[ a_1(T_5 - T_1) + \frac{a_2}{2} (T_5^2 - T_1^2) + \frac{a_3}{3} (T_5^3 - T_1^3) + \frac{a_4}{4} (T_5^4 - T_1^4) + \frac{a_5}{5} (T_5^5 - T_1^5) \right]$$

Because $C_p$ and $C_v$ are dependent on temperature, the adiabatic exponent $k = C_p / C_v$ will also vary with temperature. Therefore, the equation often used for a reversible adiabatic process with constant $k$ cannot be used for a reversible adiabatic process with a variable $k$. In the present work, the fourth order polynomial of the specific heat will be used to
find a relation between $T_1$ and $T_2$ as well as $T_3$ and $T_4$. The equation for the reversible adiabatic process with variable specific heat can be written as follows:

$$S_j - S_i = 0 = \int_a^b C_v \frac{dT}{T} + R \ln \left(\frac{V_j}{V_i}\right)$$

(12)

during the isentropic processes 1 to 2 and 3 to 4 Eq. (10) will be used to get a relation between $T_1$ and $T_2$ and another one between $T_3$ and $T_4$. Therefore, after substituting the value for $C_v$ from Eq. (4) and performing the integration of Eq. (12), equations describing processes 1 → 2 and 3 → 4 can be respectively expressed as follows

$$(a_1 - 1) \ln \left(\frac{T_2}{T_1}\right) + a_2(T_2 - T_1) + \frac{a_3}{2}(T_2^2 - T_1^2) + \frac{a_4}{3}(T_2^3 - T_1^3) + \frac{a_5}{4}(T_2^4 - T_1^4) = \ln(r_c)$$

(13)

$$(a_1 - 1) \ln \left(\frac{T_3}{T_4}\right) + a_2(T_3 - T_4) + \frac{a_3}{2}(T_3^2 - T_4^2) + \frac{a_4}{3}(T_3^3 - T_4^3) + \frac{a_5}{4}(T_3^4 - T_4^4) = \ln(r_e)$$

(14)

The compression, $r_c$, and expansion, $r_e$, ratios are defined as

$$r_c = \frac{V_1}{V_2} \quad \text{and} \quad r_e = \frac{V_4}{V_3} = \frac{V_5}{V_2}$$

(15)

For an ideal Miller cycle model, there are no heat-transfer losses. However, for a real Miller cycle, heat-transfer irreversibility between the working fluid and the cylinder wall is not negligible. It is assumed that the heat loss through the cylinder wall is proportional to the average temperature of the working fluid and the cylinder wall, and that, during the steady state operation, the wall temperature remains approximately invariant. The heat added to the working fluid by combustion is given by the following linear-relation [9,13,15,19,23]

$$Q_{in} = M[A - B(T_2 + T_3)]$$

(16)

where $A$ and $B$ are constants related to the combustion and heat-transfer processes.

Taking into account the friction loss of the piston [10,17], and assuming a dissipation term resulting from the friction force as being a linear function of the velocity, then

$$f_\mu = -\mu v = -\mu \frac{dx}{dt}$$

(17)

where $\mu$ is the coefficient of friction and $x$ is the piston displacement. Then, the lost power is

$$P_\mu = \frac{dW_\mu}{dt} = -\mu \frac{dx}{dt} \frac{dx}{dt} = -\mu v^2$$

(18)

The piston’s mean-velocity is

$$\bar{v} = \frac{x_5 - x_2}{\Delta t_{5\to 2}} = \frac{x_2(r_e - 1)}{\Delta t_{5\to 2}}$$

(19)

Thus the lost power can be written as

$$P_\mu = b_1(r_e - 1)^2$$

(20)

where $x_2$ is the piston’s position corresponding to the minimum volume of the trapped gases, $\Delta t_{5\to 2}$ is the time spent in the power stroke and $b_1 = \frac{\mu x_5}{(\Delta t_{5\to 2})^2}$. Thus, the power output $P_{output}$ can be calculated from Eqs. (3), (7), (9), (10) and (20).

$$P_{output} = \frac{W}{\tau} - P_\mu$$

(21)
Where $W$ is the net work of the cycle ($W=Q_{in}-Q_{out1}-Q_{out2}$). Then the $P_{output}$ can be expressed as

$$P_{output} = \frac{-\left[ (a_1-1)T_f + T_s + \frac{a_2}{2}T_s^2 + \frac{a_3}{3}T_s^3 + \frac{a_4}{4}T_s^4 + \frac{a_5}{5}T_s^5 \right]}{C_1(T_3 - T_s) + C_2(T_2 - T_s) + C_3(T_1 - T_s)} - b_1(r_e - 1)^2$$

(22)

The efficiency of the cycle $\eta_{th}$ can be calculated from the $P_{output}$ and Eqs. (3) and (7) as $\eta_{th} = \frac{P_{output}}{Q_{in}}$ which can be simplified as shown in Eq. (23)

$$\eta_{th} = 1 - \frac{-\left[ (a_1-1)T_f + T_s + \frac{a_2}{2}T_s^2 + \frac{a_3}{3}T_s^3 + \frac{a_4}{4}T_s^4 + \frac{a_5}{5}T_s^5 \right]}{(a_1-1)(T_3 - T_s) + \frac{a_2}{2}(T_3^2 - T_s^2) + \frac{a_3}{3}(T_3^3 - T_s^3) + \frac{a_4}{4}(T_3^4 - T_s^4) + \frac{a_5}{5}(T_3^5 - T_s^5)} - MR\left[ (a_1 - 1)(T_3 - T_s) + \frac{a_2}{2}(T_3^2 - T_s^2) + \frac{a_3}{3}(T_3^3 - T_s^3) + \frac{a_4}{4}(T_3^4 - T_s^4) + \frac{a_5}{5}(T_3^5 - T_s^5) \right]$$

(23)

When the values of $r_C$, $r_e$ and $T_1$ are given, $T_2$ can be obtained from Eq. (13); then, substituting from Eq. (7) into Eq. (16) yields $T_3$, and $T_4$ can be found using Eq. (14). The last unknown is $T_5$, which can be deduced from the entropy change of an ideal-gas equation: first; the entropy change $\Delta S_{3 \rightarrow 2}$ between states 2 and 3, is equal to the entropy $\Delta S_{4 \rightarrow 3}$ change between states 4 and 1. Thus

$$\Delta S_{3 \rightarrow 2} = \Delta S_{4 \rightarrow 1} = \Delta S_{4 \rightarrow 5} + \Delta S_{5 \rightarrow 1}$$

(24)

$$dS = C_v \frac{dT}{T} + R \frac{dV}{V} \text{ or } dS = C_p \frac{dT}{T} - R \frac{dP}{P}$$

(25)

Processes $2 \rightarrow 3$ and $4 \rightarrow 5$ occur at constant volume and $5 \rightarrow 1$ is a constant-pressure process. By substituting the specific heat from Eqs. (4) and (5) and integrating from the initial to the final state of the process, then: the change in entropy in Eq. (24) can be written as

$$\Delta S_{4 \rightarrow 1} = R \left[ (a_1 - 1)\ln \left( \frac{T_5}{T_2} \right) + a_2(T_3 - T_s) + \frac{a_3}{2}(T_3^2 - T_s^2) + \frac{a_4}{3}(T_3^3 - T_s^3) + \frac{a_5}{4}(T_3^4 - T_s^4) \right]$$

$$\Delta S_{4 \rightarrow 5} = R \left[ (a_1 - 1)\ln \left( \frac{T_4}{T_5} \right) + a_2(T_4 - T_s) + \frac{a_3}{2}(T_4^2 - T_s^2) + \frac{a_4}{3}(T_4^3 - T_s^3) + \frac{a_5}{4}(T_4^4 - T_s^4) \right]$$

$$\Delta S_{5 \rightarrow 1} = R \left[ a_1 \ln \left( \frac{T_5}{T_1} \right) + a_2(T_5 - T_s) + \frac{a_3}{2}(T_5^2 - T_s^2) + \frac{a_4}{3}(T_5^3 - T_s^3) + \frac{a_5}{4}(T_5^4 - T_s^4) \right]$$

Substituting $T_1$, $T_2$, $T_3$ and $T_4$ into Eq. (24), we get $T_5$. Also, substituting $T_1$, $T_2$, $T_3$, $T_4$ and $T_5$ into Eqs. (22) and (23) permits the efficiency and power to be estimated. Then, the relations between the power output and the compression ratio, as well as between the thermal efficiency and the expansion ratio, of the Miller cycle can be derived.

### 3. Numerical example and discussion

The following constants and parameters have been used in this exercise: $A=60,000$ J/mol, $B=25$ J/mol K, $b_1=33$ kW, $M=1.57 \times 10^{-5}$ kmol, $T_1 = 300$ K, $k_1=0.004$ J/mol K$^2$, $b=20$ J/mol K, $a=28$ J/mol K, $C_1=8.128 \times 10^{-6}$ s/K and $C_2=18.67 \times 10^{-6}$ s/K, and $C_3=10 \times 10^{-6}$ s/K [4]. The linear model of Al-Sarkhi et al. [4] is shown in Eqs. (26) and (27)

$$C_p = a + k_1 T$$

(26)

$$C_v = b + k_1 T$$

(27)
where $a$, $b$ and $k_1$ are constants. The values of the constants are 28 J/mol K, 20 J/mol K, and 0.004 J/mol K², respectively. The constant specific heats in the constant model were taken as $C_p = 29.11$ kJ/kmol K and $C_v = 20.8$ kJ/kmol K. Cases were studied numerically for values of the expansion ratio ($r_e$) from 6 to 15. A special case that presents a comparison of the best line that fits the polynomial is performed instead of the linear model of Al-Sarkhi et al. [4]. The slope of the best straight line is $k_1 = 0.0047$ J/mol K² which makes the closest straight line to the polynomial within the temperature range used in this study. The $R$-squared value of the best linear fit is 0.98. Fig. 2 shows the models of the specific heats used in this study.

Fig. 2. Specific heats models used in this study.

Fig. 3. Effect of specific heat model on the variation of the efficiency with compression ratio (a) $r_e = 8$, (b) $r_e = 10$, and (c) $r_e = 12$. 
Figs. 3 and 4 show the effects of the temperature-dependent specific heat of the working fluid on the thermal efficiency of the cycle with heat and irreversible friction losses. The thermal efficiency versus compression-ratio characteristics are approximately exponential-like curves. The efficiency versus expansion ratio characteristics approximate to parabolic-like curves. They reflect the performance characteristics of a real irreversible Miller cycle engine.

Fig. 3 shows the effects of specific heat model used on the performance of the cycle for an expansion ratio of 8, 10 and 12. The thermal efficiency increases with increasing compression ratio. The thermal efficiency is the greatest when the polynomial model of the specific heat is used and in the case of $r_c = 8$ and, it is the maximum for the case of the linear model and $r_c = 10$, while it is the maximum for the constant specific heat for $r_c = 12$. This is due the increase or decrease of the heat rejected by the working fluid and the heat added by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid. The previously mentioned parameters are very sensitive to the specific heat equation used since this equation will be integrated along the process to get the amount of heat added or rejected by the working fluid.

Fig. 4 indicates the effects of the used specific heat model on the efficiency of the cycle for different values of the expansion ratio $r_c$. The efficiency increases with increasing expansion ratio, and reaches a maximum value and then decreases. The effect of the specific heat on the efficiency is clear in these figures.

Fig. 5 shows the comparison between the best straight line that fits the fourth order polynomial of the specific heat of the special case. The best linear fit of the polynomial has better results than the linear model but still overpredicting the thermal efficiency of the cycle especially at high expansion ratios as shown in Fig. 5. The amount of heat added to and rejected from the cycle involves an integration of the specific heat equation then the order of the equation will be
increased. Even if the difference between values of the specific heats from different models is small but after the integration operation this leads to larger differences.

According to the above analysis, it can be concluded that the effects of the temperature-dependent specific heat of the working fluid on the cycle performance are significant, and should be considered carefully in practical cycle analysis and design. The constant value of the specific heat and the linear model is either over or underpredicting the thermal efficiency. The polynomial model should be used and the simplicity of linear model and the constant value of the specific heat should not be compromised.

4. Conclusion

An air-standard Miller cycle model, assuming a temperature-dependent specific heat of the working fluid as well as heat and irreversible friction losses, has been investigated numerically. The performance characteristics of the cycle show that there are significant effects of the temperature-dependent specific heat of the working fluid. The linear and constant equation of the specific heat will have a significant impact on the thermal efficiency of the studied cycle and either over or underpredicting the efficiency of the Miller cycle compared to fourth order polynomial specific heat. The best linear model that fits the fourth order polynomial of the specific heat resulted in closer cycle efficiency to the fourth order polynomial but not an exact match. Even if a small difference in the values of specific heat for different models could exist, the exact or most accurate specific heat should be used since the heat addition and rejection processes involve an integration operation of the specific heat which in the end makes a larger difference in the values of power and efficiency. The results obtained from this research are compatible with those in the open literature, for other cycles,
and may be used with assurance to provide guidance for the analysis of the behaviour and design of practical Miller engines. Future studies should discuss the possible effects of fuel additives in order to achieve a less temperature-dependent specific heat of the working fluid.

References