An analytical model is introduced in this paper to predict the stress–strain behavior of axially loaded circular reinforced concrete (RC) columns with internal longitudinal reinforcement and lateral ties confined by carbon fiber-reinforced polymer (CFRP) composites. The proposed model is a two stage stress strain curve, with a second-order polynomial in the first stage and a linear relationship with a reduced slope in the second stage. The model is an extension of the confinement model introduced by Richart et al. for confinement concrete. The model is based on the general mechanism of confinement, compatibility and equilibrium equations, and failure criteria of CFRP composites. The main parameters considered are the CFRP volumetric ratio ($\eta_f$), column size, and concrete compressive strength ($f_c$). The results predicted by the model were in good agreement with the results of current investigation and other proposed models of CFRP-confined circular RC columns reported in the literature. In addition, based on the investigation results, new design guidelines were developed that can be effectively used to determine the ultimate confined axial stresses of a RC column.

**INTRODUCTION**

Many confinement models have been developed to predict the response of confined concrete. The majority of them were originally proposed to predict the response of concrete confined by steel stirrups or continuous sleeves [1]. Kwan et al. [2] proposed an axial and lateral stress–strain model for FRP-confined concrete. The confinement model for axially loaded concrete confined by circular fiber-reinforced polymer tubes was proposed by Fam and Rizkalla [3]. Shirmohammadi et al. [4] studied the stress–strain model for circular concrete columns confined by FRP and conventional lateral steel. The effect of confinement level, aspect ratio and concrete strength on the cyclic stress–strain behavior of FRP-confined concrete prisms was investigated by Abbasnia et al. [5]. The stress–strain behavior of FRP-confined concrete under cyclic compressive loading was investigated by Abbasnia and Holakoo [6]. The effects of confining stiffness and rupture strain on the performance of FRP-confined concrete were studied by Dong et al. [7]. Moran and Panteles [8] proposed a stress–strain model for fiber-reinforced polymer-confined concrete. The experimental and theoretical prediction of axial load capacity of concrete-filled FRP tube columns is investigated by Mohamed and Masmoudi [9]. Hany et al. [10] proposed an axial stress–strain model of carbon fiber-reinforced polymer (CFRP)-confined concrete under monotonic and cyclic loading. The experimental compressive behavior of concrete externally confined by composite jackets is investigated by Berthet et al. [11]. The axial compression experiments and plasticity modeling of FRP-confined concrete members is studied by Rousakis et al. [12]. The stress–strain model for concrete confined by FRP composites is investigated by Youssef et al. [13]. Issa and Tobaa [14] presented an analytical description to predict the confined concrete strength and corresponding strain from the unconfined concrete strength. It was shown that the minimum volumetric ratio of the transverse reinforcement required by ACI code is applicable for normal and high strength concrete with strengths up to 82.7 MPa. Harajli et al. [15] presented a comprehensive study and evaluation of existing models. According to Harajli, proposed stress–strain model parameters of FRP-confined concrete can be classified primarily into three parameters: volumetric ratio of the FRP jackets, aspect ratio of the column section, and area of longitudinal and lateral steel reinforcement. Harajli concluded that in reinforced concrete...
(RC) columns, the FRP jackets prevent premature failure of the concrete cover and buckling of the steel bars, leading to substantially improved performance. The corresponding improvements become less significant as the aspect ratio of the column section increases. The results predicted by the model showed very good agreement with the results of the current experimental program and other test data of FRP-confined circular and rectangular columns.

Inspection of the previous stress–strain relationships indicated that most of the existing models of confined concrete are predicting the enhancement in the strength of the steel-confined concrete as a function of one value of confining pressure. The models were based on the following confinement model proposed by Richart et al. [16] from testing of confined concrete specimens under hydrostatic pressure:

\[ f_{cc} = f_{co} \left(1 + k_1 \frac{f_l}{f_{co}}\right) \]  
\[ e_{cc} = e_{co} \left(1 + k_2 \frac{f_{cc}}{f_{co}} - 1\right) \]

where \( f_{cc} \) and \( e_{cc} \) are the confined concrete compressive strength and corresponding strain, respectively, \( f_{co} \) and \( e_{co} \) are the unconfined concrete compressive strength and corresponding strain, \( k_1 \) and \( k_2 \) are the coefficients of confinement effectiveness for strength and ductility, respectively, and \( f_l \) is the lateral hydrostatic pressure. The usual value of 0.002 was used for \( e_{co} \).

The Harajli et al. [15] proposed that stress–strain model is the latest design-oriented model for evaluating the effect of confinement on the axial strength of concrete columns. In Harajli expressions for FRP-confined columns, the confined concrete compressive strength, \( f_{cc} \), and the corresponding strain, \( e_{cc} \), are expressed as functions of the effective constant lateral confining pressure, \( f_l \), which itself depends on volumetric ratio of the FRP jackets, aspect ratio of the column section, and area of longitudinal and lateral steel reinforcement. The results predicted by Harajli’s model were in good agreement with the results of the current experimental program and other test data of FRP-confined circular and rectangular columns. The \( f_{co} \) and \( e_{cc} \) for circular column are expressed as follows:

\[ f_{cc} = f_{co} \left(1 + 1.25 \frac{f_l}{f_{co}}\right) \]  
\[ e_{cc} = e_{co} \left(1 + \frac{25,800e^{1.17}}{(\rho_f E_f)^{0.85}} e_t + 2.0 \right) \left(\frac{f_{cc}}{f_{co}} - 1\right) \]

where \( \rho_f \) is the volumetric ratio of FRP, \( E_f \) is the FRP modulus of elasticity, and \( e_t \) is the effective lateral strain. Several confinement effectiveness \( k_1 \) (refer to Eq. 1) and \( k_2 \) (refer to Eq. 2) models were calculated based on analytical and experimental data for FRP-confined concrete [17–21]. The stress–strain behavior of circular columns confined by FRP composites has been extensively studied [22–27]. Because the behavior of the stress–strain curve is affected by many parameters, such as volumetric ratio, concrete compressive strength, and column size effect, only few analytical models have been proposed to evaluate the stress–strain behavior [15, 27].

In this study, an analytical investigation was carried out to study the effects of column size, concrete compressive strength \( f_{cc}' \), and CFRP volumetric ratio \( \rho_f \) on the stress–strain behavior of circular confined RC columns with internal longitudinal reinforcement and lateral ties wrapped with CFRP composites. Based on the NLFEA and experimental test results of this investigation, a new stress–strain relationship and design guidelines of CFRP-confined RC columns were proposed. Finally, the accuracy of the proposed stress–strain relationship was verified with the experimental test results, Mander et al.’s model [18], and Harajli et al.’s model [15].

**DESCRIPTION OF EXPERIMENTAL AND NLFEA PROGRAM REPORTED BY ISSA ET AL. [28]**

Fifty-five small scale CFRP-confined circular RC column specimens of 150 mm in diameter were fabricated and tested to failure by Issa et al. [28]. All columns were longitudinally reinforced with 4#3 steel bars \( (\rho = 1.56\%) \) and laterally reinforced with spiral steel reinforcement, 4.75 mm in diameter, spaced at 75 mm center to center along the entire height of the columns. The spacing of the spirals of 75 mm on center was based on the volumetric spiral reinforcement ratio \( \rho_s \) as required by the ACI 318-08 Eq. 10-5 [29]. The tested CFRP-strengthened columns were strengthened with one, two, three, four, and five layers of CFRP sheets in the transverse direction. Figure 1 shows the reinforcement details, cross section, and instrumentation of the tested columns. The concrete mixture used in the fabrication of all specimens had an average cylindrical concrete compressive strength \( f_{cc}' \) of 55 MPa at the time of testing. The average yield stress of the longitudinal and spiral steel reinforcements was 410 MPa. The carbon fiber used is unidirectional in the form of two sheets, manufactured in wide strips with a tensile strength of 3,800 MPa and an elastic modulus of 230 GPa.

The parameters considered in the nonlinear finite element analysis (NLFEA) were column size, \( f_{cc}' \), and number of CFRP layers in the transverse direction, which can be expressed as CFRP volumetric ratio, \( \rho_f \). Figure 2 shows the reinforcement details, cross-section, and instrumentation of the NLFEA columns reported by Issa et al. [28]. The NLFEA columns of columns was divided into four groups depending on their section diameter of 150, 300, 450, and 600 mm. For each section diameter, four groups of columns were modeled depending on their concrete compressive strength \( f_{cc}' \) of 28, 41, 55, and 69 MPa. In

DOI 10.1002/pc
the first group, nine columns were modeled, one control specimen (without CFRP) and eight columns with different CFRP volumetric ratios ($q_f$) of 0.00433, 0.00867, 0.013, 0.01733, 0.02167, 0.026, 0.0303, and 0.03467. For the other three groups, five columns were modeled for each group, one control specimen (without CFRP) and four columns with different CFRP volumetric ratios ($q_f$) of 0.00433, 0.00867, 0.013, and 0.01733. A total of 96 columns were modeled using NLFEA [30]. The longitudinal and lateral reinforcement ratios with respect to the column cross sectional area were kept constant for all columns. All columns were strengthened with additional two layers of CFRP sheets at each end to prevent premature failure at the ends due to stress concentration. Table 1 shows the average experimental and NLFEA results at ultimate reported by Issa et al. [28].

DEVELOPMENT OF STRESS–STRAIN RELATIONSHIP MODEL

A two stage stress–strain ($f_{cc} - \varepsilon_{cc}$) relationship of CFRP-confined concrete is proposed as shown schematically in Fig. 3. In the first stage, because the lateral strains and the consequent lateral confinement pressure are small, the shape of the stress–strain relationship can be described using the ascending branch ($0 \leq \varepsilon_{cc} \leq \varepsilon_{ci}$) of the stress–strain relationship of confined concrete and can be expressed by a second-order polynomial equation similar to the one suggested by Mander et al. [18]. That is, a simple mathematical expression fits the stress–strain relationship as follows:

$$f_{cc} = A\varepsilon_{cc}^2 + B\varepsilon_{cc} + C \quad (5)$$

The ascending branch of confined concrete satisfies three boundary conditions: initial condition ($\varepsilon_{cc} = 0$ when $f_{cc} = 0$), peak condition ($\varepsilon_{cc} = \varepsilon_{ci}$ when $f_{cc} = f_{ci}$), and peak stiffness condition ($df_{cc}/d\varepsilon_{cc} = 0$ when $\varepsilon_{cc} = \varepsilon_{ci}$).

Applying these boundary conditions into Eq. 5, the first stage stress–strain relationship can be expressed as shown in Eq. 6. The linear second stage, with a reduced slope, $E_1$, can be expressed as shown in Eq. 7 similar to the equation suggested by Lam and Teng [24, 25]. The two stages, stress–strain relationship can be expressed in the following general form:

$$f_{cc} = f_{ci} \left[ - \left( \frac{\varepsilon_{cc}}{\varepsilon_{ci}} \right)^2 + 2 \left( \frac{\varepsilon_{cc}}{\varepsilon_{ci}} \right) \right] \quad \text{for} \ 0 \leq \varepsilon_{cc} \leq \varepsilon_{ci} \quad (6)$$

$$f_{cc} = f_{ci} + E_1 \left( \varepsilon_{cc} - \varepsilon_{ci} \right) \quad \text{for} \ \varepsilon_{ci} \leq \varepsilon_{cc} \leq \varepsilon_{cu} \quad (7)$$

where $f_{cc}$ and $\varepsilon_{cc}$ are the confined concrete compressive strength and corresponding strain at any point, respectively, and $f_{ci}$ and $\varepsilon_{ci}$ are the confined concrete compressive strength and corresponding strain at the interaction point between the first and second stages. According to Richart et al. [16] and Fig. 3, the $f_{ci}$, $\varepsilon_{ci}$, and $E_1$ are calculated as follows:

$$f_{ci} = f_{co} \left( 1 + k_{11} \frac{f_{ci}}{f_{co}} \right) \quad (8)$$

$$\varepsilon_{ci} = \varepsilon_{co} \left( 1 + k_{21} \frac{f_{ci}}{f_{co}} - 1 \right) \quad (9)$$

$$E_1 = \frac{f_{cu} - f_{ci}}{\varepsilon_{cu} - \varepsilon_{ci}} \quad (10)$$

where $f_{cu}$ and $\varepsilon_{cu}$ are the confined concrete compressive strength and corresponding strain at ultimate, respectively, and $k_{11}$ and $k_{21}$ are the coefficient of confinement effectiveness for strength and ductility at first stage,
respectively. According to Richart et al. [16], the ultimate confined concrete compressive strain $e_{cu}$ is calculated as

$$e_{cu} = e_{co} \left(1 + k_{22} \left(\frac{f_{ci}}{f_{co}} - 1\right) \right)$$  \hspace{1cm} (11)

where $k_{22}$ is the coefficient of confinement effectiveness for ductility at the second stage. For the general case of calculating the ultimate confined concrete compressive strength ($f_{cu}$) from vertical equilibrium of the free body diagram of the column (shown in Fig. 4), the axial force $P_o$ can be expressed in terms of the internal forces in the CFRP, steel, and concrete. Summing forces in the vertical direction yields the following axial force shown in the following equation:

$$P_o = f_i (A_k - A_{st}) + f_s A_{st} + k_f f_f A_f$$  \hspace{1cm} (12)

Where $A_k$ is the gross area of the cross section, $A_{st}$ is the total area of steel, $f_i$ is the yield strength of steel, $k_f$ is the coefficient of effectiveness of CFRP system, $f_f$ is the ultimate tensile stress of the CFRP composite, and $A_f$ is the total area of CFRP composite. Experimental studies showed that the strength predicted by Eq. 12 is too high. As a result, the compressive strength of concrete $f_{cu}$ was reduced by a factor of 0.85 according to ACI 318-08 [29]. The 0.85 factor accounts for the fact that concrete poured into forms is not as strong as concrete poured into test cylinders used to establish the stress–strain curve of concrete. Making this adjustment in Eq. 12 gives

$$P_o = 0.85f_i (A_k - A_{st}) + f_s A_{st} + k_f f_f A_f$$  \hspace{1cm} (13)

Therefore, the ultimate confined concrete compressive strength ($f_{cu}$) is expressed taking into account the stress equal to axial load divided by cross-sectional area of column as follows:

$$f_{cu} = 0.85f_i (1 - \rho_{st}) + f_s \rho_{st} + k_f f_f \rho_f$$  \hspace{1cm} (14)

where $\rho_{st}$ is the steel reinforcement ratio, $\rho_f$ is the volumetric ratio of CFRP calculated as a function of number of CFRP layer ($n_f$), $t_f$ is the thickness of CFRP sheet per

<table>
<thead>
<tr>
<th>Volumetric ratio ($\rho_f$)</th>
<th>150 mm diameter</th>
<th>300 mm diameter</th>
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<tbody>
<tr>
<td></td>
<td>$f_{cu}$ (MPa)</td>
<td>$f_{cu}$ (MPa)</td>
</tr>
<tr>
<td></td>
<td>55 NLFEA</td>
<td>55 Expr.</td>
</tr>
<tr>
<td>0.00433</td>
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<td>1.140</td>
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<tr>
<td>0.00867</td>
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<td>0.01300</td>
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<td>0.02600</td>
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</tr>
<tr>
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<tr>
<td></td>
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<td>$f_{cu}$ (MPa)</td>
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<tr>
<td></td>
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<td>55 Expr.</td>
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<td>15.773</td>
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<td>0.02600</td>
<td>2.269</td>
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<td>0.03033</td>
<td>2.780</td>
<td>4.458</td>
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<td>0.03467</td>
<td>2.855</td>
<td>4.857</td>
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<table>
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<tr>
<th>Volumetric ratio ($\rho_f$)</th>
<th>450 mm diameter</th>
<th>600 mm diameter</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$f_{cu}$ (MPa)</td>
<td>$f_{cu}$ (MPa)</td>
</tr>
<tr>
<td></td>
<td>55 NLFEA</td>
<td>55 Expr.</td>
</tr>
<tr>
<td>0.00433</td>
<td>14.417</td>
<td>14.417</td>
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<td>0.00867</td>
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</tr>
<tr>
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<td>26.384</td>
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<td>0.01733</td>
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<td>0.02167</td>
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</tr>
<tr>
<td>0.02600</td>
<td>39.195</td>
<td>39.195</td>
</tr>
</tbody>
</table>

FIG. 3. Proposed stress–strain model.
layer, and $D$ is the diameter of the cross-section as presented in the following equation:

$$\rho_f = \frac{4\pi t_f}{D} \quad (15)$$

**General Mechanism of Confinement**

At low levels of axial strain in the concrete, the transverse steel and CFRP are not significantly stressed; hence, the concrete behaves essentially as unconfined. As the axial stress increases, the corresponding lateral stress increases and the confining system develops a tensile hoop stress balanced by a uniform radial stress that reacts against the concrete lateral expansion. When the concrete stress approaches about 0.7 $f_f$ to 0.9 $f_f$, the concrete begins to increase in volume. Progressive internal micro cracking and pressure against the transverse reinforcement and CFRP sheets start to occur. When the confinement fails, the concrete core becomes unable to withstand the applied load that corresponds to a stress significantly greater than the strength of the existing concrete. As a result, rupture of the confinement thus causes a sudden failure mechanism.

The mechanics of confinement are based on two factors: the tendency of concrete to dilate and the radial stiffness of the confining system to restrain the dilation. Two conditions must always be satisfied: geometric compatibility between the core and the shell and equilibrium of forces for any segment of the confined section as shown in Fig. 5. If the confinement becomes effective before the concrete reaches its peak compressive stress, the stress increase may be substantial. On the other hand, if the CFRP sheets reach their fracture strain, they cannot provide additional confinement and the beneficial effect on the post peak behavior will be limited. One of the reasons why there is practically limited experimental or analytical information on the post peak behavior of CFRP-confined columns under uniaxial loading is the experimental difficulty in measuring the descending portion of the curve.

**Compatibility and Equilibrium Equations**

Deformation compatibility was assumed between the confining CFRP sheet and the concrete surface (perfect bond), that is, the lateral strain ($\epsilon_l$) of the confined column must be equal to the strain ($\epsilon_f$) of the CFRP composite. The lateral stress acting on the concrete provided by confinement action of the lateral spiral reinforcement and CFRP sheets can be obtained by the equilibrium of forces as illustrated in Fig. 5. The resultant lateral force ($F_l$) that is applied on the concrete can be determined using the following equation:

$$F_l = F_s + F_f \quad (16)$$

$$f_l(SD) = 2A_b f_y + 2n_t f_f f_f(S) \quad (17)$$

where $f_l$ is the lateral stress acting on concrete due to confinement, $S$ is the vertical spacing between the spirals, $A_b$ is the spiral cross-sectional area, $f_y$ is the yield strength of spirals, and $f_f$ is the tensile stress of the CFRP wrapping materials calculated as a function of the effective lateral strain ($\epsilon_l$) and modulus of elasticity of CFRP ($E_f$). From Eq. 16, the lateral stress acting on concrete can be expressed as shown in the following equation:

$$f_l = f_h + f_f = \frac{2A_b f_y}{S} + 2n_t f_f f_f(S) = \frac{2A_b f_y}{S} + \frac{2n_t f_f E_f \epsilon_l}{D} \quad (18)$$

The effective lateral strain ($\epsilon_l$) was assumed as a function of ultimate tensile strain of CFRP materials ($\epsilon_{fu}$), $\rho_f$, and $f_f$, as shown in Fig. 6. Based on the regression analysis of all the data combined, the following equation is proposed for calculating the effective lateral strain ($\epsilon_l$):

$$\epsilon_l = 0.56 \exp \left(6.12 \rho_f \sqrt{\frac{4 f_f}{f_{fu}}} \right) \epsilon_{fu} \leq 0.75 \epsilon_{fu} \quad (19)$$

where $\epsilon_l$ is the effective lateral strain, $\epsilon_{fu}$ is the ultimate tensile strain of the CFRP composite (which is 16,700 $\mu$e in this study), $\sqrt{4 f_f}$ is normalized compressive strength.
of columns with respect to normal weight concrete strength of 28 MPa, and $0.75\varepsilon_{fu}$ is the maximum effective lateral strain according to ACI Committee 440 [31].

Proposed Expressions for $k_{11}$, $k_{21}$, $k_{22}$, and $k_f$

Using the experimental and NLFEA axial stress and lateral strains, the $k_{11}$ values for the CFRP-confined specimens were estimated from Eq. 8 as a function of the confinement parameters $f_i$ and $f_f$ ($k_{11} = (f_i - f_{co})/(f_i + f_f)$) and plotted as a function of $f_i/f_{co}$ as shown in Fig. 7. In addition, for the purpose of comparison, Fig. 7 shows the predictions of the various proposed expressions for $k_{11}$. Inspection of Fig. 7 reveals that the magnitude of $k_{11}$ decreases consistently from a relatively high value of 7 in early stages of response during which the effective lateral confining pressure is low to a value close to 2.9 as the confining pressure increases. The values of $k_{11}$ models for Mander et al. [18] and Samman et al. [17] are overestimated, while for Karbhari and Gao [20] and Saafi et al. [19] $k_{11}$ models are underestimated. Based on regression analysis of all the data combined, the following equation is proposed for calculating the confinement effectiveness coefficient $k_{11}$:

$$
k_{11} = 2.5 \left( \frac{2A_{df}}{SD} + \frac{2\eta t_f E_f t_i}{D} \right) (f_{co})^{-0.4}
$$

Figures 8 and 9 shows a variation of $k_{21}$ and $k_{22}$ versus $\rho_f$ that was calculated from Eqs. 9 and 11, respectively. The results in Figs. 8 and 9 clearly show that the magnitude of $k_{21}$ and $k_{22}$ decreases with increase in $\rho_f$. Based on these observations, exact evaluation of stress–strain relationship should take into account that confinement coefficients $k_{21}$ and $k_{22}$ are not only a constant or a function of lateral strain but also a function of $\rho_f$ and $f'_{c}$ as shown in the following equations:

$$
k_{21} = 0.24 \left( \frac{f'_{c}}{4\rho_f^{0.534}} \right)
$$

FIG. 6. Variation of effective lateral strain ($\varepsilon_{li}$) with $(\rho_i \sqrt{4/f'_{c}})$. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 7. Variation of confinement effectiveness coefficient $k_{11}$ versus function of the confinement parameters and compressive strength. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 8. Variation of confinement coefficient $k_{21}$ versus CFRP volumetric ratio ($\rho_f$). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 9. Variation of confinement coefficient $k_{22}$ versus CFRP volumetric ratio ($\rho_f$). [Color figure can be viewed at wileyonlinelibrary.com]
Inspection of Figs. 8 and 9 reveals that the magnitude of $k_{21}$ and $k_{22}$ has a power relation with the $p_f$. In addition, Figs. 8 and 9 shows that the magnitudes of $k_{21}$ and $k_{22}$ are affected by the compressive strength of concrete ($f_c$) in that they increase consistently as the compressive strength increases.

Finally, the $k_f$ values for the CFRP-confined columns were estimated at ultimate from Eq. 13 as a function of the $p_f$ and the normalized compressive strength of columns with respect to normal weight concrete strength of 28 MPa as shown in Fig. 10. Inspection of Fig. 10 reveals that the magnitude of $k_f$ decreases with an increase in $p_f\sqrt{f_c}$. Based on regression analysis of all the data combined, Eq. 23 is proposed for calculating the effectiveness coefficient $k_f$ (refer to Fig. 10):

$$k_f = -0.615\ln(p_f\sqrt{f_c}) - 1.3$$

(23)

Replacing the values of $p_f$, $f_c$, $e_c$, $k_{11}$, $k_{21}$, $k_{22}$, $k_f$, and from Eqs. 15, 18, 19, and (20–23), respectively, into Eqs. 8, 9, 11, and 14 leads to the following general expressions for generating the stress–strain relationship of CFRP-confined columns:

$$f_{ci} = f_{co} + 1.25\left(\frac{2A_{cf}\epsilon}{SD} + 0.20\rho_f\epsilon_f \exp \left(6.12\rho_f \sqrt{f_c/f_{co}} \right)\right)$$

(24)

$$e_{ci} = e_{co} \left(1 + 0.24f_c^{0.534}f_{ci}/f_{co} - 1\right)$$

(25)

$$e_{cu} = e_{co} \left(1 + 0.40f_c^{0.534}f_{ci}/f_{co} - 1\right)$$

(26)

$$P_o = 0.85f_c^e(A_g - A_{st}) + f_s A_{st} + (-0.615\ln(p_f\sqrt{f_c}) - 1.3)f_fA_f$$

(27)

$$f_{cu} = 0.85f_c^e(1 - \rho_{st}) + f_s \rho_s + (-0.615\ln(p_f\sqrt{f_c}) - 1.3)f_f\rho_f$$

(28)

**Comparison of Proposed Model with Other Models**

**Prediction of $f_{ce}$** The predicted values of $f_{ce}$ by the proposed and various models and the associated percent absolute errors for specimens confined with one layer and two layers of CFRP sheets are summarized in Table 2. In predicting $f_{ce}$ values, the following properties were used: $f_{co} = 55$ MPa, $f_c = 0.165$ mm for one layer and $f_c = 0.33$ mm for two layers, $f_f = 3,800$ MPa, $A_b = 17.8$ mm$^2$, $f_s = 410$ MPa, $D = 150$ mm, and $D_{core} = 125$ mm. Table 2 presented the comparison between various models for $f_{ce}$:

- The proposed model by Karbhari and Gao [20] that relates the strength enhancement to the value of $f_f$ rather than the $(f/f_{co})$ as well as the model proposed by Saafi et al. [19] and Shirmohammadi et al. [4] are largely underestimating the strength of the confined columns wrapped with one layer. Similarly, for two layers of CFRP sheets, the models proposed by Karbhari and Gao [20] and Mander et al. [18] are largely underestimating the strength.

<table>
<thead>
<tr>
<th>Model</th>
<th>One layer of CFRP</th>
<th>Two layers of CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{co}$</td>
<td>$f_{ce}/f_{co}$</td>
</tr>
<tr>
<td>Experimental</td>
<td>88</td>
<td>1.60</td>
</tr>
<tr>
<td>Karbhari and Gao [20]</td>
<td>76</td>
<td>1.38</td>
</tr>
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<td>Miriran and Shahawy [32]</td>
<td>86</td>
<td>1.56</td>
</tr>
<tr>
<td>Miyauchi et al. [33]</td>
<td>81</td>
<td>1.47</td>
</tr>
<tr>
<td>Samaan et al. [17]</td>
<td>80</td>
<td>1.45</td>
</tr>
<tr>
<td>Toutanji [23]</td>
<td>91</td>
<td>1.65</td>
</tr>
<tr>
<td>Saafi et al. [19]</td>
<td>78</td>
<td>1.42</td>
</tr>
<tr>
<td>Mander et al. [18]</td>
<td>84</td>
<td>1.53</td>
</tr>
<tr>
<td>Haralji et al. [15]</td>
<td>93</td>
<td>1.68</td>
</tr>
<tr>
<td>Kwan et al. [2]</td>
<td>82</td>
<td>1.55</td>
</tr>
<tr>
<td>Shirmohammadi et al. [4]</td>
<td>96</td>
<td>1.75</td>
</tr>
<tr>
<td>Proposed model</td>
<td>88</td>
<td>1.60</td>
</tr>
</tbody>
</table>
TABLE 3. Comparison between various models in terms of calculating $e_{cc}$.

<table>
<thead>
<tr>
<th>Model</th>
<th>One layer of CFRP sheets</th>
<th>Two layers of CFRP sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{cc}$</td>
<td>$e_{cc}/e_{co}$</td>
</tr>
<tr>
<td>Experimental</td>
<td>0.0094</td>
<td>4.7</td>
</tr>
<tr>
<td>Karbhari and Gao [20]</td>
<td>0.0033</td>
<td>1.65</td>
</tr>
<tr>
<td>Samaan et al. [17]</td>
<td>0.0257</td>
<td>12.85</td>
</tr>
<tr>
<td>Toutanji [23]</td>
<td>0.0112</td>
<td>5.6</td>
</tr>
<tr>
<td>Saafi et al. [19]</td>
<td>0.0116</td>
<td>5.8</td>
</tr>
<tr>
<td>Mander et al. [18]</td>
<td>0.0092</td>
<td>4.6</td>
</tr>
<tr>
<td>Harajli et al. [15]</td>
<td>0.0063</td>
<td>3.15</td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.0095</td>
<td>4.75</td>
</tr>
</tbody>
</table>

- Except for the previous models [2, 18–20], almost all other models are moderately accurate in predicting the compressive strength of confined specimens with no error larger than 9% for one layer of CFRP sheet and 12% for two layers of CFRP sheets.

- The proposed model gives estimation for the peak strength compared to the other models. Among the other available models, the most favorable ones are those proposed by Mirmiran and Shahawy [32], Shirmohammadi et al. [4], and Kwan et al. [2] with an absolute error of 2% for one layer of CFRP sheet. For two layers of CFRP sheets, the models proposed by Miyauchi et al. [33] and Harajli et al. [15] had absolute errors of 2%.

Prediction of $e_{cc}$. In order to carry out a comparison between the experimental and analytical strains, the value of $e_{cc}$ must be known since the expression of $e_{cc}$ involves $e_{co}$ for almost all models. Furthermore, application of the models proposed by Samaan et al. [17] and Spoelstra and Monti [22] also requires the initial elastic modulus value of concrete ($E_{co}$) and the second slope of the stress–strain curve of concrete ($E_2$). Therefore, the usual value of 0.002 was used for $e_{co}$ and $E_{co}$ was computed as a function of the unconfined strength using the ACI-code formula [29] ($E_{co} = 4730 \sqrt{f_c}$, in MPa).

The predicted $e_{cc}$ values and the percent absolute errors for the tested confined circular columns with one and two layers of CFRP sheets are shown in Table 3. The percent absolute errors in the prediction of $e_{cc}$ were much larger (between 2 and 173% for one layer and 26% to 205% for two layers of CFRP sheets) than those prediction of $f_{cc}$ between 2 and 14% for one layer and 2 and 15% for two layers of CFRP sheets). This may be attributed to the higher accuracy required when modeling the deformation characteristics of concrete rather than simply its strength properties. The stiffness of the confining device has a remarkable influence on the strain behavior. The results in Table 3 reveal the followings:

- The models included in Samaan et al. [17] and Karbhari and Gao [20] that relate $e_{cc}/e_{co}$ to $f_{cc}/f_{co}$ are overestimating and underestimating the value of $e_{cc}$ for confinement with one layer, respectively. For two layers of CFRP sheets, the models included in Samaan et al. [17] and Saafi et al. [19] are overestimating the value of $e_{cc}$.

- Compared to other models, the models by Mander et al. [18] showed better results in predicting $e_{cc}$ with errors of 2% for confined specimens with one layer of CFRP sheets and 26% for confined specimens with two layers of CFRP sheets.

- The absolute error in predicting $e_{cc}$ by the remaining models shown in Table 3 ranges from 2 to 173% for one layer of CFRP sheets and ranges from 26 to 205% for confined specimens with two layers of CFRP sheets. The premature rupture of the CFRP influences the accuracy in the prediction of $e_{cc}$. Such influence is stronger for the prediction of $e_{cc}$ than $f_{cc}$ because of the higher sensitivity of the equations giving ($e_{cc}/e_{co}$) to the value of $f_c$ and because of the wider range of percentage variation of ($e_{cc}/e_{co}$) when compared to ($f_{cc}/f_{co}$).

- The proposed equation to predict $e_{cc}$ as a function of the ultimate confinement pressure provides a good estimate of the experimental ($e_{cc}/e_{co}$) ratio compared to those of all other available models.

Comparison of Proposed Model with the Results Reported by Issa et al. [28]

The stress–strain responses obtained from experimental test results, NLFEA, and proposed model for the columns confined with one, two, three, four, and five layers of CFRP sheets in the transverse direction are plotted as shown in Fig. 11. The results were also compared with the stress–strain models of Mander and Harajli. Nonlinear finite element analysis results exhibited good agreement with the experimental test results followed by the proposed model for one, two, three, four, and five layers of CFRP confinement. The stresses at peak and post peak of the experimental results are slightly lower (less than 1.7%) than those predicted by the proposed model and very close to the NLFEA results. The response predicted by Mander et al. [18] was in good agreement with the experimental results in the first stage. Also, it predicts higher values of stress (from 0 to 22%) after the peak than the experimental, NLFEA, and the proposed model values. This difference might be attributed to the fact that Mander et al.’s model [18] was formulated assuming a constant value of the confining pressure throughout the loading history. Conversely, CFRP behaves elastically until failure, and the inward stress increases continuously. The response predicted by Harajli et al. [15] was not in good agreement with the experimental results when compared.
with the response of columns confined with one, two, and three layers of CFRP sheets, while a remarkable agreement was observed with the columns confined with four and five layers of CFRP sheets. This variation was due to the fact that Harajli’s model was formulated for rectangular columns confined with FRP sheets rather than circular columns.

**Recommended Design Guidelines to Predict Column Strength**

This section reports the design guidelines for the use of CFRP composites for confinement of circular RC columns. For strengthening and upgrade, strengthening of concrete members with externally bonded FRP composites has received remarkable attention. FRP composites have been used as an external strengthening technique in RC columns to replace conventional steel plate and concrete jackets for several reasons. The proposed Eq. 27 for predicting the ultimate axial load capacity was applied to the columns confined with one to eight layers of CFRP sheets, since it was formulated based on the $p_f$ and $f'_c$. The ultimate load values predicted by the NLFEA are shown in Table 1 and Fig. 12. Inspection of Table 1 and Fig. 12 reveals that there is a good agreement between
the NLFEA and the predicted peak-load model. In order to include the size effect on the ultimate axial load capacity of Eq. 27, an additional strength reduction factor ($w_f$) was included. Based on the NLFEA results of all the data combined, values of $w_f$ for the CFRP-confined specimens were calculated from the reduction of strength due to size effect. To determine the reduction in strength, the new size column strength was subtracted from the original size column strength and divide by the original size column strength. The following equation is proposed for predicting the size reduction factor:

$$
\psi_f = 1 - \alpha(S_d - 1)
$$

where $\alpha$ is a constant of 0.03 obtained from regression analysis of data as shown in Fig. 13, and $S_d$ is a scale factor assumed as a function of diameter of the column,
number of CFRP layers, \( n_f \), and CFRP thickness. For all NLFEA columns, the proposed equation for calculating the scale factor \( S_d \) is given in Eq. 30. Based on the regression analysis of NLFEA results, the proposed equation for predicting the size reduction factor with coefficient of correlation of 0.995 is given in Eq. 31. Applying the values of \( \psi_f \) from Eq. 31 into Eq. 28 leads to Eq. 32:

\[
S_d = \frac{n_f}{230.77 \rho_f} \tag{30}
\]

\[
\psi_f = 1 - 0.03(S_d - 1) \tag{31}
\]

\[
f_{cu} = \psi_f[0.85f' / (1 - \rho_f) + f_y + (-0.615 \ln (\rho_f \sqrt{4f' / q}) - 1.3)f_y \rho_f] \tag{32}
\]

To predict the ultimate confined compressive strength \( f_{cu} \) by using Eq. 32, four series of columns depending on their section diameter of 150, 300, 450, and 600 mm were tested. For each section diameter, four groups of columns were modeled depending on their compressive strength \( f' \) of 28, 41, 55, and 69 MPa. For each group, nine columns were modeled, one control specimen (without CFRP) and eight columns with different \( \rho_f \) of 0.00433, 0.00867, 0.013, 0.01733, 0.02167, 0.026, 0.03033, and 0.03467. The design guidelines can be effectively utilized to determine the ultimate confined compressive strength for a certain necessary \( \rho_f \), \( f' \), and column size effect as shown in Fig. 14.

Figure 15 shows comparisons between the predicted results by Eid and Paultre [34] and Lee et al. [35] models for confined CFRP columns against the experimental results. The comparison reveals that the proposed guidelines exhibit good agreement with a slight variation in the \( f_{cu,model}/f_{cu,exp} \) (variation ranges from 1 to 18%, with an average of 3%). Therefore, the recommended design guidelines can be considered effective in providing acceptable predictions of the ultimate confined compressive strength for a certain number of CFRP sheets, \( f' \), and size of the circular column. Most of the experimental and analytical studies were conducted to study the mechanical behavior of circular RC columns confined with CFRP composites. Nevertheless, information about the effects of critical parameters, such as the CFRP volumetric ratio, size effect, and concrete compressive strength, were not fully considered, or not all integrated in one model. This paper proposed an original model that can be utilized to predict the stress–strain response of CFRP-confined circular RC columns, and recommended design guidelines that can be effectively utilized to predict the ultimate axial stress of RC columns confined with CFRP for a certain CFRP volumetric ratio, size effect, concrete compressive strength, and column size.

CONCLUSIONS

Based on the results of this investigation, the following conclusions and observations are made:

1. The results showed that: (a) the ultimate axial strength increases with increasing the \( \rho_f \) to certain limiting value, (b) the enhanced strength and ductility is more clear for lower \( f' \) columns and higher \( \rho_f \), and (c) the size effect is more obvious in smaller size column in terms of axial strength.

2. The magnitude of \( k_{11} \) decreases consistently from a relatively high value of 7 in early stages of response, during which the effective lateral confining pressure is low, to a value close to 2.9 as the confining pressure increases.

3. The magnitudes of \( k_{21} \) and \( k_{22} \) decrease with the increase in CFRP volumetric ratio (\( \rho_f \)).

4. The magnitude of \( k_f \) decreases with an increase in \( \rho_f \) and a decrease in \( f' \).

5. The proposed model for predicting the stress–strain response of circular columns confined with one to five layers of CFRP sheets in the transverse direction showed excellent agreement with the results of this investigation in terms of stress–strain behavior and ultimate state of confined column under axial loading.

6. The design guidelines can be effectively used to determine the ultimate confined compressive strength for a certain necessary number of CFRP sheets, \( f' \), and size of the circular column.

NOMENCLATURE

\( \rho_f \) CFRP volumetric ratio (\( 4nf_f / D \))
\( n \) Number of CFRP layers
\( t_f \) Thickness of CFRP sheet per layer
\( D \) Diameter of the cross-section
\( f' \) Concrete compressive strength
\( f_{cc} \) Confined concrete compressive strength
\( e_{cc} \) Confined concrete strain
\( f_{co} \) Unconfined concrete compressive strength
\( e_{co} \) Unconfined concrete strain
$k_1$ Coefficient of confinement effectiveness for strength

$k_2$ Coefficient of confinement effectiveness for ductility

$f_l$ Lateral hydrostatic pressure

$E_f$ CFRP modulus of elasticity

$\varepsilon_f$ Effective lateral strain of CFRP materials

$f_{ci}$ Confined concrete compressive strength at the interaction point between the first and second stage

$\varepsilon_{ci}$ Confined concrete strain at the interaction point between the first and second stage

$f_{cu}$ Ultimate confined concrete compressive strength

$P_o$ Ultimate axial force

$k_{11}$ Coefficient of confinement effectiveness for strength at the first stage

$k_{21}$ Coefficient of confinement effectiveness for ductility at the first stage

$k_{22}$ Coefficient of confinement effectiveness for ductility at the second stage

$A_g$ Gross area of the cross section

$A_{st}$ Total area of steel

$f_y$ Yield strength of steel

$k_f$ Coefficient of effectiveness of CFRP system

$f_f$ Ultimate tensile stress of the CFRP composite

$A_f$ Total area of CFRP composite

$F_l$ Resultant lateral force

$S$ Spacing between the spirals

$A_b$ Spiral cross-sectional area

$\varepsilon_{fu}$ Ultimate tensile strain of CFRP materials

$\nu_f$ Size effect additional strength reduction factor

$S_{st}$ Scale factor

$\rho_{st}$ Steel reinforcement ratio

REFERENCES


29. ACI 318-08. American Concrete Institute, *Building Code Requirements for Reinforced Concrete*, American Concrete Institute, Farmington Hills, MI (2008).


